Ownership structure and control in incomplete market economies with transferable utility.

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Preliminary Version

Abstract

We consider an economy with incomplete markets and a single firm and assume that utility can be freely transferred in form of the initially available good 0 (quasilinearity). In this particularly simple and transparent framework, the objective of a firm can be expressed as the maximization of the total utility of its control group $\mathcal{C}$ measured in units of good 0. We analyze how the size and the composition of $\mathcal{C}$ influence the firm's market behavior and state conditions under which the firm sells it output at prices which are at, above, or below their marginal cost levels, respectively. We discuss the assumption of competitive price perceptions and point out important differences between the concepts of a Drèze and of a Grossman-Hart equilibrium that occur in spite of the close similarity of the formulas that are used to define them.

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1 Introduction

The theory of incomplete markets deals with intertemporal economies with uncertainty about future states of the world. Tradable assets can be used to transfer wealth across time and states. However, the trading possibilities are restricted because the asset span has less than full dimension. This fact entails conceptual problems which we examine in a particularly simple setting.

We assume throughout the paper that utility can be transferred in the form of the initially available good 0. As a consequence, the objective of a firm can be defined as the maximization of the total utility sum (or welfare) of its control group measured in units of good 0. We focus on the case of transferable utility because of its transparency. Moreover, a general theory of incomplete markets with production should not fail to handle special cases appropriately.

If markets are incomplete the production activities of a publicly traded firm affect agents in different ways. We adopt the framework of a corporation in the sense of Magill and Quinzii (1996), §32, and assume that the group $\mathcal{O}$ of owners of original shares receives the net value of the firm. That is to say, every original owner receives his shares' market value and contributes to the production costs in proportion to his original shareholdings. The buyers of shares obtain the future dividends. Since dividends are state dependent share purchases provide a protection against individual risks. Full insurance is impossible in incomplete markets and the success of an investment can be associated with substantial risk.[1]

There are two time periods, $t = 0$ and $t = 1$. At $t = 0$, the economy is in state 0 and one of the states $s = 1, \dots S$ will obtain at $t = 1$. Furthermore, there is one good per state, which bears the state's name. We assume that there are finitely many firms, but we often consider the case of a single firm. Each firm j has a technology $Y_j \subset \mathbb{R}_- \times \mathbb{R}^S_+$ which allows it to convert good 0 into a state dependent output at $t = 1$. Consumer i is endowed with $\delta^i_j \geq 0$ original shares of firm j and a vector $e^i = (e^i_0, e^i_{\mathbf{1}}) \geq 0$ with $e^i_0 > 0$ of consumption goods.[2] A stock market operates at $t = 0$ where original shares are exchanged against good 0. Consumer i's final shares of firm j are denoted by $\vartheta^i_j \geq 0$. The price of all shares of firm j is denoted q_j. In a *stock market equilibrium* the price system is such that the total demand $\sum_i \vartheta^i_j$ equals 1 for every firm j. The demand for shares depends on the profile of production plans y_j and the stock prices q_j. In order to choose the production plan y_j firm j needs to have a well defined objective.

There are several reasons why consumers do, in general, not agree on the objective of a firm. First, a net seller of shares gains from a high share price whereas a net buyer loses. Second, under our convention that inputs are paid by the original owners, these owners tend to prefer lower production levels than the final owners who receive the firm's output without participating in its production cost. Third, shares of firm j are an asset whose future benefits depend on the

[1] An instructive example is Daimler-Chrysler. Looking backwards, this firm has burned tens of billions of Euros/US dollars in its short history.

[2] We follow the convention to let the subscript $\mathbf{1}$ denote the S components of a vector associated with $t = 1$.

profile of production plans. Because production decisions can influence the asset span and different consumers face different risks they do typically not agree on which production plans firms should carry out. The importance of the three effects differs across consumers according to their original endowments of shares and goods as well as their preferences. The assumption of transferable utility allows us to capture all three aspects in a particularly simple and transparent way.

Assume that firm j is controlled by some group $\mathcal{C}_j$ of consumers. The members of the control group $\mathcal{C}_j$ typically differ with respect to their original and their final shareholdings as well as their insurance needs. The ownership structure matters because different control groups tend to pursue different goals. Consider, for instance, the extreme case in which the group $\mathcal{O}_j$ of original owners of a firm never holds final shares so that $\mathcal{O}_j$ is disjoint from the group $\mathcal{F}_j$ of j's final shareholders for all production plans y_j. If $\mathcal{C}_j = \mathcal{O}_j$ and $\mathcal{O}_j \cap \mathcal{F}_j = \emptyset$ then the firm's goal is to maximize the net market value $q_j - c_j$ where c_j denotes firm j's cost. In this case, shares will typically be traded at prices which exceed marginal costs.

On the other hand, if the firm is controlled by its final rather than its original owners the production level will be high and it can very well be optimal for $\mathcal{C}_j = \mathcal{F}_j$ to let the share price q_j fall below marginal costs [cf. Section 4]. Deviations from perfectly competitive behavior are rarely considered in the literature on incomplete market economies with production and one would like to know when and why this is justified. We do not assume that firms act as price takers.

To take another extreme case, suppose $\mathcal{O}_j \subseteq \mathcal{F}_j = \mathcal{C}_j$ for all production decisions. Then the stock market price q_j ceases to play any role in the firm's objective because a redistribution of good 0 among the members of $\mathcal{C}_j$ leaves $\mathcal{C}_j$'s aggregate utility unaffected. Since q_j becomes irrelevant in the case under consideration, the firm's task is to find a balance between today's cost c_j and the future benefits of its members.

Such a balance is found in the case of a Drèze equilibrium. Drèze equilibria can be defined in various ways. In §31 of their comprehensive book, Magill and Quinzii (1996) argue that Drèze equilibria should be considered within the framework of partnership economies, which differs from the present setting in the following way. A partnership economy has constant returns to scale, there are no original shares, and production costs are borne by the final shareholders in proportion to their shares.

The concept of a *Drèze equilibrium* can be based on infinitesimal transfers of good 0 and the following first order condition: The production plan of each firm j is such that the group $\mathcal{F}_j$ of j's shareholders cannot change it infinitesimally and make infinitesimal transfers of good 0 among its members such that every $i \in \mathcal{F}_j$ makes a first order utility gain. Infinitesimal share adjustments need not be taken into account because of the envelope theorem. Observe, though, that the existence of infinitesimal utility gains is equivalent to the existence of infinitesimal utility losses at an interior stock market equilibrium. The first order approach is one reason why Drèze equilibria can be undesirable.

Although we adopt the framework of a corporation and not that of a partnership economy in the sense of §32 in Magill and Quinzii (1996), we can make the following assumption within our framework to reconcile the different settings. Assume that returns to scale are constant and that all firms are controlled by the grand coalition $\mathcal{G}$ of all consumers. Then every firm j acts as a price taker which takes the following firm specific price system as given:

$$\pi_j = \sum_{i \in \mathcal{F}_j} \vartheta^i_j DU^i(x^i) = \sum_{i \in \mathcal{G}} \vartheta^i_j DU^i(x^i), \tag{1}$$

where x^i is i's equilibrium consumption, and $DU^i(x^i)$ is i's utility gradient normalized such that the partial derivative with respect to today's consumption equals 1. Price taking behavior with respect to (1) provides an alternative characterization of a Drèze equilibrium [cf. Magill and Quinzii (1996), §31].

Equation (1) illustrates the lack of unanimity due to market incompleteness given that the members of $\mathcal{G}$ agree that the firm should act as a price taker so that profits play no role because of constant returns to scale. If markets are incomplete there are no budget hyperplanes that make the individual utility gradients $DU_i(x_i)$ point into the same direction. If every final shareholder is seeking to maximize profits with respect to some price system, then shareholder i would like the firm to maximize profits with respect to $\pi_i = DU_i(x_i)$. The price system in (1) presents a compromise between different final shareholders.

The reason why the firm acts as a price taker in a Drèze equilibrium has nothing to do with competition among firms. It is also achieved in the case of a monopolist and complete markets because it rests on the inclusive nature of the control group. A firm that is controlled by a group containing at least all its customers has no reason to introduce a distortion that harms the control group without extracting wealth from anybody else. This point becomes obvious if the group equals the grand coalition $\mathcal{G}$ and there is nobody else. It tends to break down as soon as some customers are not included in the control group.

The second equality in equation (1) is a pure tautology because $\vartheta^i_j = 0$ for every $i \in \mathcal{G} \setminus \mathcal{F}_j$. However, there are reasons to focus on $\mathcal{G}$ rather than on $\mathcal{F}_j$. Drèze (1974) aims at constrained efficiency of the whole economy. Consider a planner who cannot split assets to alleviate the market incompleteness but who can choose production plans, allocate shareholdings, and distribute the total endowments of good 0. An allocation is *constrained efficient* if this planner cannot make every consumer better off. Drèze equilibria can be characterized by the first order condition for constrained efficiency. If a stock market equilibrium is constrained efficient it must be a Drèze equilibrium.

A social welfare maximum is a particularly desirable Drèze equilibrium for the following reason. The common control group $\mathcal{G}$ coordinates the production decisions of all firms j, whereas Drèze equilibria can suffer from a lack of coordination across firms [cf. Drèze (1974)]. Moreover, even if there is only one firm in the economy, Drèze equilibria suffer from the fact that higher order utility changes are disregarded. Interior minima and maxima of $\mathcal{G}$'s aggregate utility are both Drèze equilibria.

We turn to the equilibria of Grossman and Hart (1979). In a GH equilibrium, every firm j maximizes profits with respect to the price system

$$\pi_j = \sum_{i \in \mathcal{O}_j} \delta_j^i DU^i(x^i). \tag{2}$$

The striking similarity between (1) and (2) suggests a close similarity between the cases in which the initial and the final shareholders control the firm. However, this viewpoint is deceptive.

The size of the control group $\mathcal{O}_j$ is given by the distribution of initial shares and not endogenously determined. The original shares can be assigned to $\mathcal{G} \setminus \mathcal{F}_j$. In this case, firm j has no reason to act as a price taker unless it is forced to do so by competition.

Grossman and Hart (1979) write on p. 299 f.: "We are making the assumption of "utility taking,"as opposed to price taking, behavior. ... If the firm is competitive then it will assume that the prices of state contingent incomes are not affected by its action." The underlying idea is that "consumer i's consumption makes a small contribution to his overall utility."

Thus, GH have a specific setting in their minds, which plays no role in the understanding of Drèze equilibria. However, they do not provide an explicit model so that it is difficult to see which assumptions have to be made to provide an appropriate basis for their approach.

GH explain the reason why final shares play the same role in equation (1) as the initial shares do in equation (2) by the fact that the manager of the firm is acting in the interest of the final shareholders in the first case and in the interest of the initial shareholders in the second. This leaves the question of what the manager does if $\mathcal{F}_j$ and $\mathcal{O}_j$ coincide unanswered. The concept of a Drèze equilibrium has the advantage that it does not rely on any perceptions.

Furthermore, if returns to scale are constant and firms act as price takers then profits vanish at every stock market equilibrium. The only motive that $\mathcal{O}_j$ can have in this case is to insure its members against bad states at $t = 1$. This motive is captured by $\mathcal{O}_j$'s final and not by its original shares.

In Section 3, we present a simple, numerical example. We consider the case of a single firm and drop the index j. Thus, we do not try to model a situation which GH had in their minds. Our conclusions are, however, to some extent independent of the supply side. If the manager of the firm acts in $\mathcal{O}$'s interest by maximizing $\mathcal{O}$'s total utility then the firm's goal is independent of how the original shares δ^i are allocated across $\mathcal{O}$ as long as the support of the distribution does not change. We find it puzzling that the δ^i's are the important variables in (2) if their distribution leaves $\mathcal{O}$'s welfare invariant.

The paper is organized as follows. In Section 2, we describe our framework and focus on the Drèze rule. Grossman-Hart equilibria are discussed in Section 3. In Section 4 we analyze the conflict between original and final shareholders and its impact on market power. Section 5 deals with competitive price perceptions and Section 6 concludes.

2 Social welfare maximization and the Drèze rule

If markets are complete and perfectly competitive it is irrelevant how the production sector is organized. The production sector can be represented by a single firm which possesses the economy's aggregate production set as its technology. This firm and all individual firms which it represents pursue a common goal, the maximization of profits with respect to a price system which coordinates all production decisions.

If markets are incomplete then different firms can have different control groups with different objectives and the coordination of production decisions breaks down. From a normative perspective, it is desirable to investigate the benchmark case in which all firms are governed by the same group, the grand coalition $\mathcal{G}$. Since we assume that utility is transferable, $\mathcal{G}$'s goal is to maximize the aggregate utility of all consumers subject to the technological constraints in the whole economy.

To keep matters as simple as possible we explore now the following case. There is a single firm, every consumer holds at least a tiny fraction of original shares, and the firm is controlled by the group $\mathcal{O}$ of original shareholders. The group of final shareholders can be arbitrarily small.

We give a formal description of the framework. As usual, the subscript 0 refers to $s = 0$ and the subscript $\mathbf{1}$ refers to all states $s = 1, \cdots, S$ at $t = 1$. A consumption vector is written as $x = (x_0, x_{\mathbf{1}}) \in \mathbb{R}_+^{S+1}$. Every utility function takes the form $U^i(x_0, x_{\mathbf{1}}) = x_0 + V^i(x_{\mathbf{1}})$. The initial endowment of consumer i is $e_i = (e_0^i, e_{\mathbf{1}}^i)$, where e_0^i is a (sufficiently large) positive number. For convenience, we will set $e_{\mathbf{1}}^i = 0$ in specific examples. As a consequence, there is no need for short sales.

There is a single firm so that we can drop the index j. The only asset in the economy consists of shares in this firm.[3] The firm chooses an output vector $y_{\mathbf{1}}$ and incurs a cost of $c(y_{\mathbf{1}})$ units of good 0. The production vector $y = (y_0, y_{\mathbf{1}})$ with $y_0 = -c(y_{\mathbf{1}})$ lies on the efficient boundary of Y.

The cost $c(y_{\mathbf{1}})$ is carried by the original shareholders in proportion to their initial shares δ^i. The final shares after trade on the stock market are denoted by $\vartheta_i(y_{\mathbf{1}})$. The firm's value $q(y_{\mathbf{1}})$ is determined by a market clearing condition. At a *stock market equilibrium*, $q(y_{\mathbf{1}})$ is such that the total demand for shares $\sum_{i\in\mathcal{G}} \vartheta_i(y_{\mathbf{1}})$ equals 1. A priori, it is *not* ruled out that the firm possesses market power although firms under the Drèze rule (1) or the GH rule (2) act as price takers.

If the firm produces $y_{\mathbf{1}}$ then i consumes $x^i(y_{\mathbf{1}}) = (e_0^i + (\delta^i - \vartheta^i(y_{\mathbf{1}}))\, q(y_{\mathbf{1}}) -$

[3] It is not difficult, though, to describe two or more publicly traded firms in a similar way as long as they are controlled by the same group $\mathcal{G}$.

$\delta^i c(y_{\mathbf{1}}), e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}})$. The welfare of any control group $\mathcal{C}$ of consumers equals

$$W^{\mathcal{C}}(y_{\mathbf{1}}) = e_0^{\mathcal{C}} + (\delta^{\mathcal{C}} - \vartheta^{\mathcal{C}}(y_{\mathbf{1}}))q(y_{\mathbf{1}}) - \delta^{\mathcal{C}} c(y_{\mathbf{1}}) + \sum_{i \in \mathcal{C}} V^i(e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}), \quad (3)$$

where $e_0^{\mathcal{C}}$ is $\mathcal{C}$' initial endowment of good 0 and $\delta^{\mathcal{C}}$ its endowment of original shares. The firm aims to maximize the welfare of its control group.

Proposition 1. *Assume that the monopolistic firm is controlled by the group $\mathcal{O}$ of original owners and that $\mathcal{O} = \mathcal{G}$. If $y_{\mathbf{1}}$ maximizes $\mathcal{O}$'s welfare then $y_{\mathbf{1}}$ maximizes profits given the price system*

$$\pi(y_{\mathbf{1}}) = \sum_{i \in \mathcal{O}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i) = \sum_{i \in \mathcal{F}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i), \quad (4)$$

where x^i is i's optimal consumption bundle. The optimal bundle $y_{\mathbf{1}}$ is sold at marginal costs, that is to say, $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

Observe that the firm uses the Drèze rule although it is not controlled by its final shareholders. It says that the marginal costs paid today equal the marginal benefits consumed tomorrow. Consumer i's marginal benefits are proportional to i's final shares $\vartheta^i(y_{\mathbf{1}})$.

Proof. If $\mathcal{G} = \mathcal{O}$ instructs the firm to produce some bundle $y_{\mathbf{1}}$ then $\mathcal{O}$'s consumption of good 0 becomes $e_0^{\mathcal{G}} - c(y_{\mathbf{1}})$. The market value $q(y_{\mathbf{1}})$ is irrelevant for $\mathcal{G}$'s consumption at $t = 0$ since the members of $\mathcal{G}$ pay and receive $q(y_{\mathbf{1}})$ and the utility functions are quasilinear. The original shares play no role since $\mathcal{G}$'s aggregate utility does not depend on how $q(y_{\mathbf{1}})$ and $c(y_{\mathbf{1}})$ are allocated. The firm maximizes

$$W^{\mathcal{G}}(y_{\mathbf{1}}) = e_0^{\mathcal{G}} - c(y_{\mathbf{1}}) + \sum_{i \in \mathcal{G}} V^i(e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}).$$

Let $v^i(y_{\mathbf{1}}) = V^i(e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}})$. The first order condition for welfare maximization can be stated as $Dc(y_{\mathbf{1}}) = \sum_{i \in \mathcal{G}} Dv^i(y_{\mathbf{1}}) = \sum_{i \in \mathcal{G}} \vartheta^i(y_{\mathbf{1}}) DV^i(x^i_{\mathbf{1}})$ where $x^i = (x^i_0, x^i_{\mathbf{1}}) = (e^i_0 + (\delta^i - \vartheta^i(y_{\mathbf{1}}))\, q(y_{\mathbf{1}}) - \delta^i c(y_{\mathbf{1}}), e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}})$ denotes i's optimal consumption. If the firm maximizes $W^{\mathcal{G}}(y_{\mathbf{1}})$ then it maximizes profits with respect to marginal cost prices $\pi(y_{\mathbf{1}}) = \sum_{i \in \mathcal{G}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i)$.

It remains to show that $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. Because consumer i maximizes his utility given the market price $q(y_{\mathbf{1}})$ when he buys shares, i's utility gradient at the optimum is orthogonal to the ray through $(-q(y_{\mathbf{1}}), y_{\mathbf{1}})$ along which agents trade. Thus, $(1, DV^i(x^i_{\mathbf{1}}))(-q(y_{\mathbf{1}}), y_{\mathbf{1}}) = 0$. All final shareholders i attribute the same value $DV^i(x^i_{\mathbf{1}})y_{\mathbf{1}} = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = q(y_{\mathbf{1}})$ to $y_{\mathbf{1}}$. $\square$

It is remarkable that the distribution of original shares plays no role other than to ensure that every consumer belongs to $\mathcal{O}$. In this case, there is no profit motive. The firm acting on behalf of $\mathcal{C} = \mathcal{G}$ does not suffer from a loss of market power due to competition. Rather it behaves as a monopolist who must not harm its control group $\mathcal{G}$ by raising the price above marginal costs. This reasoning is in line with Drèze (1974) who uses the first order condition for constrained efficiency.

The welfare maximum constitutes a Drèze equilibrium that coincides with a GH equilibrium only if the original shares δ^i happen to coincide with the final shares ϑ^i. The Drèze rule presents a benchmark case if not all original or not all final shareholders belong to the control group. Both cases are investigated in Section 4.

The fact that the group $\mathcal{F}$ of final shareholders also appears in (4) is a byproduct which should not be misinterpreted. If $\mathcal{F} \subsetneqq \mathcal{O}$ and $\mathcal{C} = \mathcal{F}$ then $\mathcal{F}$ will typically not let the firm act according to (4) [see Proposition 3 in Section 4]. The reason is that we have not adopted the framework of a partnership economy and the original shares are an obligation to contribute to the production cost. Thus, $\mathcal{C} = \mathcal{F}$ does not take the production costs fully into account if $\mathcal{F} \subsetneqq \mathcal{O}$.

The argument used above to derive the Drèze rule can also be applied in the following case. Let $\mathcal{C} \supseteq \mathcal{F} \cup \mathcal{O}$ so that $\mathcal{C}$ pays and receives q. Then the maximization of $\mathcal{C}$'s welfare $W^{\mathcal{C}}(y_{\mathbf{1}}) = e_0^{\mathcal{C}} - c(y_{\mathbf{1}}) + \sum_{i \in \mathcal{C}} v^i(y_{\mathbf{1}})$ leads to the first order condition $Dc(y_{\mathbf{1}}) = \sum_{i \in \mathcal{C}} \vartheta^i(y_{\mathbf{1}}) DV^i(x_{\mathbf{1}}^i) = \sum_{i \in \mathcal{F}} \vartheta^i(y_{\mathbf{1}}) DV^i(x_{\mathbf{1}}^i)$.

Remark . *Whenever the control group contains $\mathcal{O} \cup \mathcal{F}$, its welfare is independent of q. The firm acts as a price taker even if it faces no competition.*

3 Grossman-Hart equilibria in a quasilinear example

In the example, there are two types $\tau = 1, 2$ of consumers, N^i persons of each type and a single firm. We do not aim to provide a framework with monopolistic competition in the spirit of Grossman and Hart (1979) but want to explain some difficulties arising in the present simple setting.

A consumer is denoted by $i = (\tau, n)$ where τ is i's type and the number $n \in \{1, \cdots, N^i\}$ serves to distinguish consumers of the same type. For convenience, we use the following terminology. Consumers of the same type have the same preferences and initial endowments of goods. However, they may differ with respect to their original shares which can be varied parametrically. S equals 2 and the utility functions defined on $\mathbb{R}^3_{++}$ are given by

$$\begin{aligned} U^1(x_0, x_1, x_2) &= x_0 + 2\log(x_1) + \log(x_2), \\ U^2(x_0, x_1, x_2) &= x_0 + \log(x_1) + 2\log(x_2), \end{aligned} \tag{5}$$

respectively. Every consumer has the initial endowment $(e_0, 0, 0)$.

We assume that the costs to produce (y_1, y_2) are $c(y_1, y_2) = y_1^r + y_2^r$, where the scale elasticity $r \geq 1$. This allows us to consider constant and strictly decreasing returns to scale.

We determine the asset demand for both types and the market clearing asset price. Assume that the firm's output equals (y_1, y_2). Shares of (y_1, y_2) can be bought on the stock market. A consumer i of type 1 who decides to buy the share ϑ^i consumes the bundle $(e_0^i + \delta^i(q-c) - \vartheta^i q,\ \vartheta^i y_1,\ \vartheta^i y_2)$ and obtains the utility $e_0^i + \delta^i(q-c) - \vartheta^i q + 2\log(\vartheta^i y_1) + \log(\vartheta^i y_2)$. The utility maximizing amount of ϑ^i is obtained if $-q + 2/\vartheta^i + 1/\vartheta^i = 0$. Therefore, the demand for shares of a consumer of type 1 is given by $\vartheta^i = 3/q$. Due to the symmetry of the types, the demand for shares of a consumer of type 2 also equals $\vartheta^i = 3/q$. Since there are $N = N_1 + N_2$ consumers, market clearing requires $\sum^i \vartheta^i = 3N/q = 1$. The firm's market value is $q = 3N$.

The fact that q is constant is remarkable for the following reason. According to the assumption of competitive price perceptions in Grossman and Hart (1979), the original shareholders, who do not know the function q, feel that small output changes induce linear changes of q. A shareholder i with the quasilinear utility function U^i uses his utility gradient $DU^i = (1, DV^i)$ at his optimal consumption plan to evaluate the change. In the example, the assumption of competitive price perceptions is violated everywhere for every shareholder. We will discuss competitive price perceptions in Section 5.

Competitive price perceptions have undesirable consequences. In the quasilinear case, *a redistribution of original shares within $\mathcal{O}$ is irrelevant for $\mathcal{O}$'s welfare.* However, the original shares δ^i play a decisive role in a GH equilibrium where the firm maximizes profits with respect to the price system $\sum_{i\in\mathcal{O}} \delta^i DU^i(x^i)$. In contrast to the Drèze rule, the GH rule is not oriented towards welfare and constrained efficiency.

An important difference between final and original shares is the following. *Final shareholdings are chosen by economic agents, whereas original shareholdings can be assigned arbitrarily.* In our example, this fact has the following implication. Consider two economies, $\mathcal{E}$ and $\tilde{\mathcal{E}}$. In economy $\mathcal{E}$, the Drèze rule (1) is used whereas the GH rule (2) is applied in economy $\tilde{\mathcal{E}}$. There are N^i consumers of type i in $\mathcal{E}$ and $\tilde{N}^i$ in $\tilde{\mathcal{E}}$. The economies are of the same size, that is to say, $N_1 + N_2 = \tilde{N}_1 + \tilde{N}_2 = N$. We have $\mathcal{G} = \mathcal{F} = \mathcal{O}$ in both economies so that price taking behavior is well founded.

Proposition 2. *The original shares in $\tilde{\mathcal{E}}$ can be assigned in such a way that the unique Grossman-Hart equilibrium of $\tilde{\mathcal{E}}$ coincides with the unique Drèze equilibrium of $\mathcal{E}$. In the Grossman-Hart equilibrium of $\tilde{\mathcal{E}}$, the firm aims to maximize the social welfare in $\mathcal{E}$.*

Proof. To prove the claim, consider $\mathcal{E}$ and assume that the output bundle (y_1, y_2) is produced. The utility gradient of a consumer of type 1 is $DU^1(x_0, x_1, x_2) = (1, 2/x_1, 1/x_2)$ and that of a consumer of type 2 is $DU^2(x_0, x_1, x_2) = (1, 1/x_1, 2/x_2)$. Since the Drèze rule (1) is used in $\mathcal{E}$ and the consumption at $t = 1$ is the same for all consumers, that is to say $(x_1, x_2) = (y_1/N, y_2/N)$, the firm maximizes profits with respect to

$$\pi = \frac{N_1}{N}\left(1, \frac{2N}{y_1}, \frac{N}{y_2}\right) + \frac{N_2}{N}\left(1, \frac{N}{y_1}, \frac{2N}{y_2}\right) = \left(1, \frac{2N_1 + N_2}{y_1}, \frac{N_1 + 2N_2}{y_2}\right). \quad (6)$$

In equilibrium, π equals the technology gradient $(1, ry_1^{r-1}, ry_2^{r-1})$. Hence, the equilibrium output is $y_1 = ((2N_1 + N_2)/r)^{1/r}$ and $y_2 = ((N_1 + 2N_2)/r)^{1/r}$. The output is independent of how the original shares are distributed because the utility functions are quasilinear.

Consider now economy $\tilde{\mathcal{E}}$, which implements a Grossman-Hart equilibrium, and assign to each of the $\tilde{N}^i$ consumers of type i the original shares $\delta^i = N^i/(\tilde{N}^i N)$. Then the weight of the utility gradient of type i in the GH rule (2) is $\tilde{N}^i \delta^i = N^i/N$. Therefore, the firm maximizes profits with respect to the price system π as given by (6) and we obtain the same equilibrium production. $\square$

What matters for the welfare of $\mathcal{O}$ is $\mathcal{O}$'s size and composition. However, our example illustrates the following problem. The size of $\mathcal{O}$ can be changed without any effect on the stock market by redistributing original shares within types. For each type $i = 1, 2$, let $\bar{\delta}^i$ be the total number of original shares held by all consumers of type i and denote the normalized utility gradient of any such consumer by π^i. Then the GH rule (2) becomes

$$\pi = \bar{\delta}_1 \pi_1 + \bar{\delta}_2 \pi_2. \tag{7}$$

The degree of concentration of the original shares cannot be deduced from (7). Suppose that all original shares are in the hands of one consumer of each type and that $\mathcal{C} = \mathcal{O}$. Then the firm should charge prices above marginal costs. In order to justify price taking behavior one needs a widely spread distribution of original shares or a sufficient degree of competition among firms. In the next section we analyze how the distribution of original and final shares influences the stock market price. In the first part the firm is controlled by final, in the second part by the original shareholders.

4 The conflict between final and original shareholders.

We have shown that the Drèze rule (1) results if the distributional conflicts at $t = 0$ are internalized by a sufficiently large control group such as $\mathcal{G}$.[4] However, an original shareholder $i \notin \mathcal{F}$ receives $\delta^i(q(\mathbf{y_1}) - c(\mathbf{y_1}))$ units of good 0 whereas a final shareholder $i \notin \mathcal{O}$ pays $\vartheta^i(\mathbf{y_1})q(\mathbf{y_1})$ and neglects the costs when he can influence the choice of $\mathbf{y_1}$. We argue that the market price $q(\mathbf{y_1})$ tends to *fall below* $\mathbf{y_1}$'s value at marginal cost prices if the corporation is controlled by $\mathcal{F}$.

For simplicity's sake, we assume that there are consumers who always buy shares and others who never do so. That is to say, $\mathcal{C} = \mathcal{F}$ is supposed to be independent of the choice of the production plan $\mathbf{y_1}$. Deviations from the Drèze rule occur if there are original owners who do not belong to $\mathcal{C}$.

[4]There is also no conflict at $t = 0$ in a partnership economy because there are no original owners.

We analyze these deviations. If the corporation produces $\mathbf{y_1}$ then i consumes $(e_0^i + (\delta^i - \vartheta^i(\mathbf{y_1}))\, q(\mathbf{y_1}) - \delta^i c(\mathbf{y_1}), e_{\mathbf{1}}^i + \vartheta^i(\mathbf{y_1})\mathbf{y_1})$. Let $\delta^{\mathcal{F}}$ denote the total amount of original shares owned by $\mathcal{F}$ and $e_0^{\mathcal{F}}$ its initial endowment of good 0. Then $\mathcal{F}$'s welfare equals

$$W^{\mathcal{F}}(\mathbf{y_1}) = e_0^{\mathcal{F}} + (\delta^{\mathcal{F}} - 1)q(\mathbf{y_1}) - \delta^{\mathcal{F}} c(\mathbf{y_1}) + \sum_{i \in \mathcal{F}} v^i(\mathbf{y_1}), \tag{8}$$

where $v^i(\mathbf{y_1}) = V^i(e_{\mathbf{1}}^i + \vartheta^i(\mathbf{y_1})\mathbf{y_1})$. We obtain the first order condition

$$(1 - \delta^{\mathcal{F}})Dq(\mathbf{y_1}) + \delta^{\mathcal{F}} Dc(\mathbf{y_1}) = \sum_{i \in \mathcal{F}} \vartheta^i(\mathbf{y_1}) DV^i(x_{\mathbf{1}}^i), \tag{9}$$

where $x_{\mathbf{1}}^i$ is i's optimal consumption at $t = 1$.

As shown at the end of the proof of Proposition 1, $DV^i(x_{\mathbf{1}}^i)\mathbf{y_1} = q(\mathbf{y_1})$ for every $i \in \mathcal{F}$. That is to say, if i uses his utility gradient to evaluate $\mathbf{y_1}$ then the resulting value $DV^i(x_{\mathbf{1}}^i)\mathbf{y_1}$ coincides with $\mathbf{y_1}$'s market value $q(\mathbf{y_1})$.

If the firm produces $\mathbf{y_1}$ and the associated technology gradient is $\pi(\mathbf{y_1}) = (1, \pi_{\mathbf{1}}(\mathbf{y_1}))$ then $\pi_{\mathbf{1}}(\mathbf{y_1}) = Dc(\mathbf{y_1})$. Therefore, if we use (9) to evaluate $\mathbf{y_1}$ we obtain the following relationship between the market value $q(\mathbf{y_1})$ and $\mathbf{y_1}$'s value $\pi_{\mathbf{1}}(\mathbf{y_1})\mathbf{y_1}$ at marginal cost prices:

$$(1 - \delta^{\mathcal{F}})Dq(\mathbf{y_1})\mathbf{y_1} + \delta^{\mathcal{F}} Dc(\mathbf{y_1})\mathbf{y_1} = q(\mathbf{y_1}). \tag{10}$$

In the numerical example in Section 3, $q(\mathbf{y_1})$ is constant so that equation (10) reduces to $\delta^{\mathcal{F}}\pi_{\mathbf{1}}(\mathbf{y_1})\mathbf{y_1} = q(\mathbf{y_1})$. If $\delta^{\mathcal{F}} = 1$ we obtain a Drèze equilibrium and $\mathbf{y_1}$ is sold at marginal costs. However, if $\delta^{\mathcal{F}}$ falls below 1 then $\pi_{\mathbf{1}}(\mathbf{y_1})\mathbf{y_1} = q(\mathbf{y_1})/\delta^{\mathcal{F}}$ exceeds $q(\mathbf{y_1})$.

This observation can be generalized as follows. Consider a given output vector $\mathbf{y_1} \gg 0$ and vary the scale λ of production. We assume that an infinitesimal increase of λ decreases the profit, that is to say, $\partial_\lambda(q(\lambda\mathbf{y_1}) - c(\lambda\mathbf{y_1}))|_{\lambda=1} < 0$. Hence,

$$\partial_\lambda q(\lambda\mathbf{y_1})|_{\lambda=1} = Dq(\mathbf{y_1})\mathbf{y_1} < \partial_\lambda c(\lambda\mathbf{y_1})|_{\lambda=1} = Dc(\mathbf{y_1}))\mathbf{y_1}\,. \tag{11}$$

Then we conclude from (10) that

$$q(\mathbf{y_1}) < (1 - \delta^{\mathcal{F}})Dc(\mathbf{y_1})\mathbf{y_1} + \delta^{\mathcal{F}} Dc(\mathbf{y_1})\mathbf{y_1} = \pi_{\mathbf{1}}(\mathbf{y_1})\mathbf{y_1}. \tag{12}$$

Competitive pricing in the sense of Magill and Quinzii (1996), p.382, means that q is a linear function. Thus $\partial_\lambda q(\lambda\mathbf{y_1})|_{\lambda=1} = 1$ and the assumption $\partial_\lambda(q(\lambda\mathbf{y_1}) - c(\lambda\mathbf{y_1}))|_{\lambda=1} < 0$ is satisfied if $\partial_\lambda(c(\lambda\mathbf{y_1}) > 1$.

Proposition 3. *If* $\partial_\lambda(q(\lambda\mathbf{y_1}) - c(\lambda\mathbf{y_1}))|_{\lambda=1} < 0$ *and* $\mathcal{C} = \mathcal{F} \supsetneqq \mathcal{O}$ *then the market value* $q(\mathbf{y_1})$ *is* below *its value* $\pi_{\mathbf{1}}(\mathbf{y_1})\mathbf{y_1}$ *at marginal cost prices.*

The question of who controls the firm and the question of who has the power to exploit whom are intrinsically related. If the firm maximizes $\mathcal{F}$'s welfare and there are original shareholders outside $\mathcal{F}$ then these shareholders are exploited in the sense that the production level is so large that their profit income falls below the perfectly competitive level.

Now we assume that the market power rests with $\mathcal{O}$ and argue that the market price $q(\boldsymbol{y_1})$ tends to *rise above* $\boldsymbol{y_1}$'s value at marginal cost prices $\pi_{\mathbf{1}}(\boldsymbol{y_1})$. Let $\vartheta^{\mathcal{O}}$ denote the total amount of final shares owned by $\mathcal{O}$ and $e_0^{\mathcal{O}}$ its initial endowment of good 0. Then $\mathcal{O}$'s welfare equals

$$W^{\mathcal{O}}(\boldsymbol{y_1}) = e_0^{\mathcal{O}} + (1 - \vartheta^{\mathcal{O}}(\boldsymbol{y_1}))q(\boldsymbol{y_1}) - c(\boldsymbol{y_1}) + \sum_{i\in\mathcal{O}} V^i(e^i_{\mathbf{1}} + \vartheta^i(\boldsymbol{y_1})\boldsymbol{y_1}) \tag{13}$$

and we obtain the first order condition

$$(\vartheta^{\mathcal{O}}(\boldsymbol{y_1}) - 1)Dq(\boldsymbol{y_1}) + Dc(\boldsymbol{y_1}) = \sum_{i\in\mathcal{O}} \vartheta^i(\boldsymbol{y_1})DV^i(x^i_{\mathbf{1}}). \tag{14}$$

If we take the inner product with $\boldsymbol{y_1}$ we obtain

$$(\vartheta^{\mathcal{O}}(\boldsymbol{y_1}) - 1)Dq(\boldsymbol{y_1})\boldsymbol{y_1} + \pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1} = \sum_{i\in\mathcal{O}} \vartheta^i(\boldsymbol{y_1})q(\boldsymbol{y_1}) = \vartheta^{\mathcal{O}}(\boldsymbol{y_1})q(\boldsymbol{y_1}). \tag{15}$$

Consider the extreme case in which no member of $\mathcal{O}$ wants to hold final shares so that $\vartheta^{\mathcal{O}}(\boldsymbol{y_1}) = 0$. Then (13) becomes $W^{\mathcal{O}}(\boldsymbol{y_1}) = e_0^{\mathcal{O}} + q(\boldsymbol{y_1}) - c(\boldsymbol{y_1})$ and the firm aims to maximize its net market value.

In our example, Dq vanishes and (15) becomes $\pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1} = \vartheta^{\mathcal{O}}q(\boldsymbol{y_1}) \le q(\boldsymbol{y_1})$, where the inequality is strict provided $\vartheta^{\mathcal{O}} < 1$. In this case, $\boldsymbol{y_1}$ is sold with a mark-up.

We assume now that $\partial_\lambda q(\lambda\boldsymbol{y_1})/q(\lambda\boldsymbol{y_1})|_{\lambda=1} < 1$. That is to say, q grows by less than 1% if $\boldsymbol{y_1}$ is increased by 1% and the boundary case of competitive pricing is ruled out. Equation (15) is equivalent to

$$q(\boldsymbol{y_1}) - \pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1} = (\vartheta^{\mathcal{O}}(\boldsymbol{y_1}) - 1)(Dq(\boldsymbol{y_1})\boldsymbol{y_1} - q(\boldsymbol{y_1})). \tag{16}$$

Because $\vartheta^{\mathcal{O}}(\boldsymbol{y_1}) < 1$ and $Dq(\boldsymbol{y_1})\boldsymbol{y_1} = \partial_\lambda(q(\lambda\boldsymbol{y_1}) < q(\boldsymbol{y_1}))$ by assumption we obtain that (16) is positive, that is to say $q(\boldsymbol{y_1}) > \pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1}$.

Proposition 4. *If $\partial_\lambda(q(\lambda\boldsymbol{y_1})/q(\lambda\boldsymbol{y_1})|_{\lambda=1} < 1$ and $\mathcal{C} = \mathcal{O} \supsetneqq \mathcal{F}$ then $q(\boldsymbol{y_1}) > \pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1}$.*

To summarize, if the control group is so large that it contains $\mathcal{O}$ and $\mathcal{F}$, for instance if $\mathcal{C} = \mathcal{G}$, then $\mathcal{C}$'s welfare takes its maximum at a Drèze equilibrium and $q(\boldsymbol{y_1}) = \pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1}$. However, $q(\boldsymbol{y_1})$ can be below or above $\pi_{\mathbf{1}}(\boldsymbol{y_1})\boldsymbol{y_1}$. The first case arises if the firm is controlled by $\mathcal{F}$, the second if it is controlled by $\mathcal{O}$.

5 Competitive price perceptions

The assumption of competitive price perceptions says that every individual $i \in \mathcal{O}$ uses his own utility gradient $DV^i(x^i_{\mathbf{1}})$ at his optimal consumption given $y_{\mathbf{1}}$ to evaluate alternative production plans. Thus, i thinks that $q(y'_{\mathbf{1}}) = DV^i(x^i_{\mathbf{1}})y'_{\mathbf{1}}$.

If transfers of good 0 are used to enable the winners of a potential change $\Delta y_{\mathbf{1}}$ of $y_{\mathbf{1}}$ to compensate the losers one faces the following difficulty. Since markets are incomplete $DV^i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$ will typically be positive for some consumers and negative for others. The first group feels that the share price will go up while the other group feels that it will go down if production is changed by $\Delta y_{\mathbf{1}}$. If $\mathcal{O}$ knows how to transfer good 0 from the members of the first group to the members of the second group then $\mathcal{O}$ must be informed about which member has which characteristics. In particular, *it is known within $\mathcal{O}$ that the individual price perceptions $q(y'_{\mathbf{1}}) = DV^i(x^i_{\mathbf{1}})y'_{\mathbf{1}}$ of $\mathcal{O}$'s members are incompatible with each other.*

The assumption of competitive price perceptions serves the following purpose. Suppose the firm changes its output slightly from $y_{\mathbf{1}}$ to $\hat{y}_{\mathbf{1}} = y_{\mathbf{1}} + \Delta y_{\mathbf{1}}$. In the quasilinear case, i's utility at $y_{\mathbf{1}}$ is $e^i_0 + \delta^i[q(y_{\mathbf{1}}) - c(y_{\mathbf{1}})] - \vartheta^i q(y_{\mathbf{1}}) + V^i(e^i_{\mathbf{1}} + \vartheta^i y_{\mathbf{1}})$. The output change $\Delta y_{\mathbf{1}}$ induces the first order utility change

$$\Delta U^i = \delta^i[Dq(y_{\mathbf{1}}) - Dc(y_{\mathbf{1}})]\Delta y_{\mathbf{1}} - \vartheta^i(y_{\mathbf{1}})[Dq(y_{\mathbf{1}}) - DV^i(x^i_{\mathbf{1}})]\Delta y_{\mathbf{1}}, \tag{17}$$

where $x^i_{\mathbf{1}} = e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}$. Consumer i, who does not know the function q, feels that, for any $\Delta y_{\mathbf{1}}$, the utility change $DV_i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$ at $t = 1$ is exactly offset by the associated price change $Dq(y_{\mathbf{1}})\Delta y_{\mathbf{1}}$. That is to say, i feels that the second bracket in equation (17) vanishes.

Competitive price perception have three important consequences for the conflict among the original shareholders. First, they assume away their conflict as final shareholders because they annihilate the second bracket in (17). Second, they create a new conflict among the members of $\mathcal{O}$ in their role as original owners since the objective market value $q(y_{\mathbf{1}} + \Delta y_{\mathbf{1}})$ is replaced by a family of subjective perceptions $DV_i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$. Third, whenever a firm has market power it is deprived of this power by the perceptions of its shareholders. That is to say, in equilibrium we have $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. In the present setting this equality follows immediately from the first order condition (16), because the right hand side of this equation vanishes by the definition of competitive price perceptions and is equal to the left hand side $q(y_{\mathbf{1}}) - \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

To derive the GH rule (2) in the present framework we rewrite the first order condition (14) as follows:

$$Dq(y_{\mathbf{1}}) - Dc(y_{\mathbf{1}}) = \sum_{i \in \mathcal{O}} \vartheta^i(y_{\mathbf{1}})(Dq(y_{\mathbf{1}}) - DV^i(x^i_{\mathbf{1}})). \tag{18}$$

Under competitive price perceptions, the left hand side of equation (18) equals $\sum_{i \in \mathcal{O}} \vartheta^i(y_{\mathbf{1}})DV^i(x^i_{\mathbf{1}}) - Dc(y_{\mathbf{1}}) = 0$ and the right hand side vanishes. Therefore, we obtain $\pi_{\mathbf{1}}(y_{\mathbf{1}}) = Dc(y_{\mathbf{1}}) = \sum_{i \in \mathcal{O}} \vartheta^i(y_{\mathbf{1}})DV^i(x^i_{\mathbf{1}})$.

Drèze equilibria have the merit that they do not rely on perceptions. The control group is so large that it wants the firm to act as a price taker. Thus, the firm needs to know the value $q(y_\mathbf{1})$ but not the function q. In GH, the firm can be controlled by a small group, which would gain by selling $y_\mathbf{1}$ at a price above $\pi_\mathbf{1}(y_\mathbf{1})y_\mathbf{1}$.

We ignore the question of why the original shareholders instruct their firm to act as a perfectly competitive price taker and ask whether they might, under suitable circumstances, have information that is more relevant for their welfare than the distribution of original shares given that non-competitive outcomes are ruled out.

Consider the case of a technology with constant scale elasticity $r \geq 1$ as in our example and fix some cost level c_0. Then all $y_\mathbf{1}$ with $c(y_\mathbf{1}) = c_0$ yield the same profit $(r-1)\,c_0$ computed with respect to the normalized technology gradients. Among these bundles the shareholders prefer the bundle $y_\mathbf{1}$ which constitutes their optimal output mix. A computation shows that the optimal output proportions are independent of the scale of production in our example.

$\mathcal{O}$ can instruct the firm to implement the optimal output mix at the competitive production level. This can be achieved without knowledge of the function q. Furthermore, this behavior appears perfectly natural in the case of constant returns to scale, i.e. $r = 1$, in which the profit is identically equal to 0 whenever the firm takes some price system as given. The procedure can also be used if r is larger than 1 and the benefits derived from $\vartheta^{\mathcal{O}}$ are substantial.

In our specific example, the procedure entails that the original shareholders maximize their welfare subject to the constraint of price taking behavior. The reason is that $q(y_\mathbf{1}) = \pi_\mathbf{1}(y_\mathbf{1})y_\mathbf{1}$ for all $y_\mathbf{1}$ with $c(y_\mathbf{1}) = c^*$ where c^* is the equilibrium cost level.

If one wants to base the objective of a firm on the original shares then these shares should be derived from economic decisions. Suppose that every consumer i had a chance to acquire original shares at period $t = -1$ and that i knows that his interests will be included in the firm's objective at $t = 0$ provided $\vartheta^i > 0$. Then every i has an incentive to possess at least one zillionth of these shares. In this case, the firm is controlled by $\mathcal{O} = \mathcal{G}$ and $\vartheta^{\mathcal{O}} = 1$. Then q drops out of the objective function (13) and the problem disappears. Similarly, in a more general framework without transferable utility, the control group $\mathcal{O} = \mathcal{G}$ will implement the Drèze equilibrium rule without any need for perceptions.

6 Conclusions

The theory of incomplete markets with production has been based on the principle that shareholders can make transfers when they decide on a production plan. This allows the winners of a potential change to compensate the losers provided the gains are sufficiently large. For simplicity's sake, we assume that good 0 can be used to transfer utility freely between consumers. Hence, the firm's objective can be defined as the maximization of a function, the welfare or total utility of its control group $\mathcal{C}$.

In the case of transferable utility, the welfare of the whole group $\mathcal{C}$ rather than each individual utility gain or loss matter. This fact has important implications. Assume that the firm is controlled by $\mathcal{O}$. If $\mathcal{C} = \mathcal{O}$ is so large that it contains all final shareholders then the market value q does not enter $\mathcal{C}$'s welfare although all its individual members can be affected by q. As a consequence, there is no need for price perceptions in this case and $\mathcal{O}$'s welfare maximum is attained at a Drèze equilibrium and not at a GH equilibrium.

Furthermore, a redistribution of the original shares δ^i within $\mathcal{C} = \mathcal{O}$ does not have any effect on $\mathcal{O}$'s welfare as long as nobody loses all his original shares so that $\mathcal{O}$ shrinks. Therefore, the weights δ^i of the utility gradients in the definition of a GH equilibrium do not enter the firm's objective in the transferable utility case beyond the fact that they determine the members of $\mathcal{O}$. The fact that the welfare neutral original shares serve as weights of the utility gradients in a GH equilibrium is a pure consequence of the shareholders' perceptions of their problem.

The assumption of competitive price perceptions combines two aspects. The first, competitive pricing, says that the stock market value q is supposed to be a linear function. Second, each consumer i feels that $q(y_{\mathbf{1}})$ coincides with i's marginal utility evaluation $DV^i(x^i)y_{\mathbf{1}}$ of $y_{\mathbf{1}}$. This implies that $y_{\mathbf{1}}$ is sold at marginal costs independently of whether this lies in $\mathcal{C}$'s genuine interest.

We have presented a numerical example in which no original shareholder satisfies the assumption of competitive price perceptions for any production decision of the firm. In the example, the function q is constant rather than linear.

The question of whether the firm should be priced at, above, or below marginal costs depends on the ownership structure and control in the following way. If the firm is controlled by its *original shareholders* and $\mathcal{F}$ is not fully contained in $\mathcal{O}$ then $\mathcal{C}$ has an incentive to sell its stock at a price $q(y_{\mathbf{1}})$ *above* marginal costs $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

However, if the firm is controlled by its *final shareholders* and $\mathcal{O}$ is not contained in $\mathcal{F}$ then $\mathcal{C}$ has an incentive to charge a price *below* marginal costs since all costs are borne by the original shareholders in our setting so that a positive fraction of the costs is not accounted for by $\mathcal{C}$.

The Drèze rule results if $\mathcal{C} \supseteq (\mathcal{F} \cup \mathcal{O})$ and $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. The case of $\mathcal{C} = \mathcal{G}$ is particularly important from a welfare perspective because every consumer's interest is taken into account. If $\mathcal{C} = \mathcal{F}$ is smaller than $\mathcal{G}$ then those consumers who would hold final shares at an alternative production plan are ignored.[5]

The last three paragraphs shed light on the need for price perceptions. Assume it is optimal for $\mathcal{C} = \mathcal{O}$ to sell its output at marginal costs although the firm is not forced to do so by fierce competition. Then $\mathcal{O}$ must contain $\mathcal{F}$. Therefore, $\mathcal{C}$ does not need to know q and $\mathcal{O}$'s welfare optimum is a Drèze equilibrium.

[5]The Drèze rule is typically used in the framework of a partnership economy without initial shares. In this case, the Drèze rule relies on the condition $C \supseteq \mathcal{F}$.

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