Where to Live?
Wages, Community Composition and Public Goods *

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Abstract We analyze the group composition problem that arises when wages depend on a profile of different types of labor and consumers have different tastes for public goods. The interaction between the distribution of tastes and labor skills leads to a stratification of the society, which has important socio-economic implications in public policy (production, tax system and inequality). We provide an equilibrium model with local public goods and collaborative private production, where heterogeneous communities are compatible with a context of anonymous crowding. Entrepreneurs guarantee efficiency by offering the policy package that best fits with the consumers' desired pattern of jurisdiction composition.

JEL Classification: C62, D71, H4, R1.

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1 Introduction

This paper addresses an important and long standing question in public policy: how the individuals' decision of where to live is related to the wages they are paid in the private production of their jurisdiction and the happiness obtained from the use of its public goods (schools, hospitals, clean air, ...).[1] [2] An immediate consequence of the individuals' decision making is the stratification of the society into different patterns of communities, which has important socioeconomic consequences for public policy, e.g. production, tax system and inequality[3]. Although some normative and empirical analyses on this issue have already been done (for instance, Bayer et al. [7], Benabou [8] and [9], Roback [33], Glaeser and Mare [18] and Glaeser and Sainz [19][4]), up to our knowledge, there is not a satisfactory equilibrium framework that analyzes this economy from both descriptive and normative perspectives.

We insert this analysis into the existing literature of local public goods. The closest paper is Konishi [23], who models a Tiebout economy where consumers sort into jurisdictions because their offer of public goods and the exogenous wage linked to each jurisdiction type. Our main departure from Konishi is that we endogenize the production technology (and thus the wages) and make it dependent on the profile of worker types in a jurisdiction. This further step is crucial for the study of the different possible patterns of community composition. The idea is that if the private production in a jurisdiction is collaborative[5] and pays high wages, then the individuals will prefer to group into a heterogeneous

[1]See Tiebout [40, p.418] for the question of what variables influence the individual's choice of municipality.

[2]The comparison between the public and private sectors is in our days an issue subject to a profound debate in the economic policy arena. Two different conceptions dominate: more versus less market oriented economies, e.g. New York (USA) versus Helsinki (Finland), respectively.

[3]For instance, see Freeman [16] and Mookherjee and Ray [30] for the inequality within a community.

[4]Glaeser and Saiz [19] provide an empirical justification of the fact that skilled cities are growing because they are becoming more economically productive (relative to less skilled cities), not because these cities are becoming more attractive places to live.

[5]The production technology is collaborative if the production requires more than one type of agent (labor input) in order to result in a positive output, i.e. it exhibits *enhancing skills*. Recall the standard definition of enhancing skills, which requires the marginal productivity of one type of inputs to be increasing in the amount of other input. Typical examples of a collaborative production are the Leontieff and Cobb-Douglas technologies.

community such that the pattern of community composition fits with the labor complementarities required in the production sector. Moreover, the existence of an heterogeneous community is compatible with the standard assumption of *anonymous crowding* in the consumption of public goods: individuals care only about the level of congestion of the public goods and not about the identities of the other individuals that make use of them.

Our model introduces new aspects in the literature of local public goods. We model the private goods production to be done within jurisdictions, so that wages are both type and jurisdiction specific. This treatment of the production differs from Wooders [41], who makes the production only dependent on the size of the jurisdiction. Also this assumption enriches the model by bringing it closer to reality, since the jurisdiction-type specific wage rate results in different wages among the different individuals of the jurisdiction. However, in order to impose such assumption, we must prevent the workers to commute among jurisdictions in order to profit from wage differentials. That is, we allow for free mobility for the individuals in their decision of which jurisdiction they want to live, but once the decision is taken they must work in the existing industry of their own jurisdiction.

Our treatment of firm organization is close to Zame [42], who integrates firm theory into a general equilibrium framework *a la* Arrow and Debreu [4]. The main difference from Zame [42] is that we want to focus on the trade-off that consumers face when choosing their place of residence (and work): wages versus public goods. For this, we integrate the theory of local public goods and firm theory. Thus, we remove one of the last remaining original Tiebout's assumption: "no restriction due to employment opportunity". We do not introduce any contractual problems into the model (skills are observable and adverse selection problems are ruled out). Our attention is primarily oriented towards the properties of societal stratification.

Our treatment of the group composition problem differs from the previous literature of jurisdiction/club formation. Alesina and La Ferrara [1], Conley and Wooders [11] and Ellickson et al. [15] addressed the issue of the individuals' participation in heterogeneous communities, but in a different context. They consider an economy where the consumers have preferences on the other types of consumers (referred as *non-anonymous crowding* in a context of local public goods) and this is the final source of heterogeneity in their models. A well known result (Scotchmer and Wooders [36]) is that in a public good economy where the consumers only care about the level of congestion (referred as *anonymous crowding*), the jurisdictions result in homogeneous communities (see also Konishi [23][6]). Our paper models jurisdictions with both local public goods (subject to *anonymous crowding*) and private collaborative production. The source of heterogeneity in the community is a consequence of the wages obtained by the consumers in the collaborative production and not because of the consumers' preferences on the other types of consumers with whom they share the public goods.

In a recent paper, Konishi [22] addresses the issue of existence of heterogeneous clubs in a context of anonymous crowding. However, even if the result seems similar to ours, the source of heterogeneity is totally different. There, mixed clubs result and are efficient if clubs have multiple facilities (e.g. gym and swimming pool) with economies of scope, whereas in the present paper heterogenous communities arise due to the labor complemetarities in the private production process.

The issue of whether heterogenous communities would become possible in an environment of anonymous crowding was first addressed by Berglas [10]. He showed that it is the interaction between the distribution of tastes and labor skills that leads to different patterns of mixed communities.[7] This work motivated important subsequent contributions in the topic of the formation of mixed communities.[8] In particular, McGuire [29] builds a concise diagrammatic analysis of the group composition problem, including conditions under which Tiebout configurations dominate Berglas-groups. However, both Berglas [10] and McGuire [29] addressed a purely normative analysis, but left aside the issue of existence of equilibrium. Those models suffer several shortcomings that prevent the study of existence of equilibrium. In particular, the approach through differential techniques is not appropriate when considering the population as a finite and discrete

[6]Konishi's [23] purpose is to prove that the Tiebout's solution to the Samuelson's [34] free rider problem holds in equilibrium, and for that he assumes that the Jurisdiction Managers only have information on the distribution of consumers' preferences. There, the jurisdictions result homogeneously populated as a consequence of the imposition of a zoning constraint that makes crowding effects anonymous.

[7]This contrasts with the well known result that identical individuals tend to concentrate in homogenous communities if the economy exhibits anonymous crowding (see Schotchmer and Wooders [36]).

[8]See Bartolome [6], Benabou [9], McGuire [29] and Schwab and Oates [37].

set. This well known "integer problem" was first analyzed by Pauly [32] and concisely summarized by Starret [39].

Our objective here is to analyze the group composition problem that arises when wages depend on a profile of different types of labor and consumers have different tastes for the public goods. Section 2 starts with an example. Section 3 establishes the model. Section 4 gives the main results: existence of equilibrium, efficiency, and conditions for heterogeneous communities. The Appendix is reserved for the proofs.

2 The Example

Let us begin with an example of the group composition problem: enhancing skills imply that individuals want to form heterogeneous communities with different types of labor skills in order to achieve high wages in the private production sector. On the other hand, individuals with the same tastes for public goods want to sort themselves into homogenous communities in order to consume their most preferred public goods. The interaction between the distribution of tastes and skills leads to different patterns of community composition. We want to compare several types of jurisdictions. For that, we take as given the supply of the different jurisdictions types. Later, in the Section 3, we model the offer of jurisdiction types by entrepreneurs.

`COMMUNITIES AND PUBLIC GOODS:` Let the economy be composed by 10 consumers of type 1 and 10 consumers of type 2. The number of consumers of type $\theta = 1, 2$ in a jurisdiction is denoted by n_θ. There is only one perfectly divisible commodity used for consumption, which price is 1. The endowment of a type 1 consumer is $e^1 = 3$ units of the commodity, while a type 2 consumer is endowed with $e^2 = 1.5$ units of the commodity. There are two available public projects: an airport (g_1) and a subway (g_2). Individuals of different types have different tastes on the two public goods, and therefore, different utilities associated to those public goods. The cost of both public projects is the same: 20 units of the commodity. In order to focus our attention on the utility derived from each public good for each type of consumer we fix the level of congestion of the public goods. We do this by fixing the number of consumers that form a jurisdiction to 10. Of course, in the model below the level of congestion is a variable of choice for the players.

PRIVATE PRODUCTION SECTOR: There are two possible private production technologies available to a jurisdiction. One of them is a Cobb-Douglas production function of the form $y^c = 10n_1^{0.5}n_2^{0.5}$. This production is collaborative in the sense that it is not possible to produce a positive amount of output by using only one type of inputs (also referred as the production exhibiting enhancing skills). Let the type θ consumer's wage if the production is y^c be denoted by α_θ^c. Then the associated profit to $y^c(n_1, n_2)$ is $10n_1^{0.5}n_2^{0.5} - \alpha_1^c n_1 + \alpha_2^c n_2$. The competitive wages that make zero profits for this technology are $\alpha_1^c = 5\left(n_2/n_1\right)^{0.5}$ and $\alpha_2^c = 5\left(n_1/n_2\right)^{0.5}$. Observe that the community that results in equilibrium is the one formed by the same number of type 1 and type 2 consumers. Otherwise, some consumers would block the coalition and improve by grouping themselves into a community with $n_1 = n_2$. Since we fixed the level of congestion in a jurisdiction to 10 consumers, we have that the optimal crowding profile is $(n_1, n_2) = (5, 5)$. Then, the competitive wages become $\alpha_1^c = 5$ and $\alpha_2^c = 5$. Next, let us consider the following bounded CRS private production technology with perfect substitutes labor inputs: $y^s = 3n_1 + 0.5n_2$, with $n_\theta \leq 10$. This technology leads to homogeneous communities if consumers of different types prefer different public goods (since the production is not collaborative). The corresponding competitive wages are $\alpha_1^c = 3$ and $\alpha_2^c = 0.5$.

UTILITIES: In order to fix the population of a community to 10 individuals, we assume that $u_\theta(x, g, n) = -\infty$ if $n \neq 10$. Otherwise, the following specification of the utility function holds. The utility of both types of consumers is separable: $u_\theta(x, g, n) = u_\theta\left(x\right) + \frac{1}{n}u_\theta\left(g\right), \theta = 1, 2$. We assume that $u_1\left(x\right) = u_2\left(x\right) = x \in \mathbb{R}_+$, so all consumers have the same preference for the consumption of the private good (later in the model this assumption is dispensed). Also let $\frac{1}{n}u_1\left(g_1\right) = \frac{1}{n}u_2\left(g_2\right) = 60/10$ and $\frac{1}{n}u_1\left(g_2\right) = \frac{1}{n}u_2\left(g_1\right) = 20/10$, (where $n = 10$), meaning that the type 1 consumer prefers the airport, whereas the type 2 consumer prefers the subway.

SAMUELSONIAN LUMP SUM TAXES: In order to cover the cost of providing the public good, type θ consumers are charged a non-anonymous Samuelsonian lump sum tax $t_\theta\left(g, (n_1, n_2)\right)$. The following expressions hold:[9] $t_1\left(g_1, (5, 5)\right) = t_2\left(g_2, (5, 5)\right) = (6/8)(20/5) = 3$ and $t_1\left(g_1, (10, 0)\right) = t_2\left(g_2, (0, 10)\right) = 20/10$.

[9] Observe that the ratio $\frac{6}{8}$ refers to the marginal contribution of a type θ_1 consumer when the public good is g_1. The marginal contribution of a type θ_2 consumer with the public good g_1 is $\frac{2}{8}$. The ratio $\frac{20}{5}$ refers to the cost of the public good divided by the number of consumers of type θ.

`INDIVIDUAL'S OPTIMIZATION PROBLEM:` The maximization problem that a type $\theta = 1, 2$ consumer faces in a jurisdiction type ω is $Max_{\{x\}}\ u_\theta(x) + \frac{1}{\bar{n}} u_\theta(\bar{g})$ subject to $x + \bar{t}^\theta_\omega \leq \bar{\alpha}^\theta_\omega + e^\theta$. The following table depicts the indirect utilities for each class of consumer under the relevant jurisdiction types. The other possible jurisdiction types obviously result in a lower payoff and, therefore, are omitted in the table below. Also observe that the payoff of jurisdiction ω_3 is only relevant for type 1 consumers, since the population of the jurisdiction is only composed of consumers of this type. In that case $(U_2(\omega_3))$ we write n.a. (does not apply). The same reasoning applies to the jurisdiction type ω_4.

Jurisdiction types	U_1	U_2
$\omega_1 = (y^c, g_1 \text{ and } (n_1, n_2) = (5, 5))$	11	6.5
$\omega_2 = (y^c, g_2 \text{ and } (n_1, n_2) = (5, 5))$	8	9.5
$\omega_3 = (y^s, g_1 \text{ and } n_1 = 10)$	10	n.a.
$\omega_4 = (y^s, g_2 \text{ and } n_2 = 10)$	n.a.	6

`NONCOOPERATIVE GAME:` Given the above payoffs, the following two players Nash game holds:

$1\backslash 2$	ω_1	ω_2	ω_4
ω_1	**11, 6.5**	0, 0	0, 6
ω_2	0, 0	8, 9.5	0, 6
ω_3	10, 0	10, 0	**10, 6**

There are two Nash equilibrium: (ω_1, ω_1) and (ω_3, ω_4).[10] Observe that the former (ω_1, ω_1) gives (strictly) higher payoffs than the latter (ω_3, ω_4). The Pareto

[10] The logic behind the table follows by doing comparative-statics. First, we show that, although type 2 consumers would obtain a higher utility under the project ω_2, this type of jurisdiction would never result in equilibrium. To see this, note that type 1 consumers prefer the jurisdiction type ω_3 to ω_2, and the formation of ω_3 is feasible for type 1 consumers since it does not require other types of consumes for the jurisdiction to be formed. Thus, ω_3 is a better alternative for type 1 consumers than ω_2. But this implies that ω_2 is not feasible since it requires half of its population being type 1 consumers. This problem does not occur for the type 2 consumers when comparing jurisdiction ω_4 (a feasible jurisdiction for type 2 consumers since it only consists of consumers of their own type) with jurisdiction ω_1, since ω_1 gives a higher payoff for these type 2 consumers than ω_4. Thus, we can conclude that jurisdiction type ω_1 is the one formed in equilibrium.

superior equilibrium jurisdiction is ω_1 : it offers the public good g_1 and has a crowding profile of 5 individuals of each type, so that the proportions satisfy the optimal proportions for the production technology y^c.

Two important points can be highlighted from our example. The first one is that, in order for a jurisdiction to be formed, the individuals must want to participate in it (in the sense that every individual type does not have a better feasible alternative) so that the specific profile of consumers' types is fulfilled (a consistency condition). The second is that, although the depicted equilibrium results to be in pure strategies, we could have used other parameter values and obtained a situation where an individual is indifferent between two (or more) jurisdiction types. That is, an equilibrium in mixed strategies seems possible at first sight. Therefore, it might be surprising to find an equilibrium in pure strategies. These two points are addressed in our model below.

3 The Model

We consider a one-period economy with two classes of agents: consumers and jurisdiction managers, the latter also referred as entrepreneurs[11] by Tiebout [40] and Mackowski [26]).

There is a finite set $\mathbf{\Theta} = \{1, ..., \Theta\}$ of agents' *crowding types* (or *external characteristics*). A *crowding type* is of a complete description of the characteristics of an agent that are relevant to the other members of his jurisdiction. In this paper these *external* characteristics take the form of labor skills and tastes for public gods, and we assume them to be observable (as Ellickson et al. [15]).[12] Consumers with the same crowding type $\theta \in \mathbf{\Theta}$ have the same labor skills and same tastes for public goods. We find this assumption convinient in order to analyze the group composition problem that arises when wages depend on a profile of different types of labor and consumers have different tastes for public goods. However, we allow consumers with the same crowding type to have different preferences on the consumption of private goods (see below).

[11] Entrepreneurship is a well established concept in public policy. For example, the city of Chicago's plan to transform its public housing is currently involved in rebuilding several sites as mixed-income communities. There, a developer uses private money to leverage private investment, builds public housing within mixed-income developments and owns the real estate.

[12] Observe that we could have gone further and associate to each observable crowding type (skills) an unobservable taste for the public goods (as Conley and Wooders [11]).

The set of consumers is a nonatomic finite measure space $(\mathbf{I}, \mathcal{I}, \upsilon)$, where I denotes this set, $\mathcal{I}$ is a σ-algebra of subsets of $\mathbf{I}$ and υ is the Lebesgue measure. The set of type θ consumers is denoted $I(\theta) \in \mathcal{I}$, with associated measure $\upsilon(I(\theta))$.

We refer to a consumer of type θ as θ_i (e.g. two consumers of the same type are differentiated by writing θ_i and $\theta_{i'}$ with $i \neq i'$). We denote the number of type θ consumers in a jurisdiction ω by $n_\omega^\theta \in \mathbb{N}_+$ (a positive natural number). The *crowding profile* of jurisdiction ω is then a vector $(n_\omega^\theta)_{\theta \in \mathbf{\Theta}}$. The number of consumers (level of congestion) in a jurisdiction ω is given by $n_\omega \overset{def}{=} \sum_{\theta \in \mathbf{\Theta}} n_\omega^\theta$. Let us define β_ω^θ as the proportion of type θ consumers in jurisdiction ω, that is, $\beta_\omega^\theta = (n_\omega^\theta / n_\omega) \in [0,1]$. We refer to $\beta_\omega = \left(\beta_\omega^\theta\right)_{\theta \in \mathbf{\Theta}}$ as the jurisdiction ω's *crowding profile of proportions.* We shall say that two jurisdictions ω and ω' have the same crowding vector of proportions if for all $\theta \in \mathbf{\Theta}$, $\beta_\omega^\theta = \beta_{\omega'}^\theta$.

We impose the following two assumptions in order to model negligible clubs with respect to the whole economy.

Assumption 1 *(Large Population): There is a continuum of consumers of each crowding type.*

Assumption 2 *(Finitely Populated Jurisdictions): Each jurisdiction ω has a finite number of consumers, that is, $\exists \hat{n}_\omega < \infty : \ n_\omega \leq \hat{n}_\omega$.*

Assumption 2 requires a finite upper bound on the number of consumers in a jurisdiction, independently of their type. Then the whole set of consumers splits into jurisdictions, which are assumed to be finitely populated. This implies that a continuum of jurisdictions must result in order to match all the consumers into jurisdictions (Ellickson et al. [15] and Konishi [22] and [23]). Wooders [41] first pointed out the need of Assumption 2, under the name of *strict small group effectiveness.*[13] Assumption 1 is standard in the literature in order to avoid integer problems that result in nonexistence of equilibrium (see Kaneko and Wooders [41]). Recently, Allouch and Wooders [2] have dispensed with Assumption 2 by assuming "Desirability of Wealth", so large political jurisdictions (such as states or countries) become possible in equilibrium. In the present paper we model small communities as entrepreneurial organizations, and thus prefer to keep the idea of macroscopically negligible jurisdictions.[14] We have in mind districts,

[13]Note that we assume a continuum of consumers of each type and hence the condition that the total population of consumers of a given type exceeds the upper bound (finite) is immediately satisfied.

[14]This is analogous to Aumann [5] pioneering concept of negligible agents.

counties and villages when referring to jurisdictions.

The set of commodities is $\mathbf{L} = \{1, ..., l, ..., L\}$. The commodity price vector is $p \in \mathbb{R}_{+}^{|L|}$, where $|L|$ denotes the cardinality of the set $\mathbf{L}$. There is a finite number of indivisible public projects, in the sense of Mas-Colell [27]. The set of public projects is denoted by $\mathbf{G} = \{1, ..., g, ..., G\}$. A public project consists of a discrete set of public goods such as schools, hospitals, parks, water supply systems, etc. In the economy there are several jurisdiction types offered by the jurisdiction managers to the consumers. A *jurisdiction type* ω is characterized by a policy package $(g_\omega, \left(n_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}, y_\omega)$, where y_ω is a private production technology (described below) , $\left(n_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}$ is the crowding profile, and g_ω is the public project. The associated level of congestion of the public good is n_ω. We denote the set of jurisdiction types by $\mathbf{\Omega} = \{1, ..., \omega, ..., \Omega\}$.

PRIVATE PRODUCTION

The private production technology y_ω maps labor inputs $\left(m_\omega^\theta\right)_{\theta\in\mathbf{\Theta}} \in \mathbb{R}_{+}^{|\Theta|}$ into private goods (outputs), i.e. $y_\omega : \left(m_\omega^\theta\right)_{\theta\in\mathbf{\Theta}} \rightarrow \mathbb{R}_{+}^{|L|}$.[15] We assume that the production technology exhibits constant returns to scale (CRS).[16] The set of available production technologies, $\mathbf{Y} = \{1, ..., y, ..., Y\}$, is a finite set.

Once a consumer has chosen a jurisdiction, he supplies his unit of labor to the private production sector of his jurisdiction. We assume that consumers with the same crowding type have the same labor skills, and, therefore, they can be seen as similar labor units. Therefore, if a jurisdiction has the crowding profile $\left(n_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}$, then there will be n_ω^θ units of labor of type θ supplied in the jurisdiction, which in turn determines the jurisdiction crowding profile of proportions β_ω. In this way, we are modeling a labor market where the offer of labor contracts, via the production entrepreneurs associated with the different juris-

[15] In fact, we could have modeled the production as a function of both labor inputs and physical capital, in which case we would write $y_\omega : (K_\omega, \left(m_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}) \rightarrow \mathbb{R}_{+}^{L}$, where K_ω is an exogenous parameter that represents the physical capita used in jurisdiction ω in the private production. But this possibility would not give us further insights on the group composition problem, and thus is ommited.

[16] It is importat to remark that if we had considered a production technology exhibiting increasing returns to scale, then two ex-ante identical consumers would choose differently in their optimization problems. This will lead to an unequal distribution of earned income among otherwise ex-ante identical agents (same type). See Freeman [16] and Mookherjee and Ray [30] for a detail study of this issue.

dictions, is accommodated to the consumers' demand for jurisdictions (inelastic labor supply).

The profits of the private production sector belonging to a jurisdiction that is characterized by a private production technology y_ω and a crowding profile $\left(n_\omega^\theta\right)_{\theta\in\Theta}$ (that turns out to be the labor inputs of this jurisdiction) are $\Pi_\omega^y = py_\omega\left(\left(n_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}\right) - \sum_{\theta\in\mathbf{\Theta}} \alpha_\omega^\theta n_\omega^\theta$, where $\alpha_\omega^\theta \in \mathbb{R}_+$ denotes the wage paid to a type θ consumer. Observe that a CRS production function implies that wages are homogeneous of degree zero in the labor inputs, which implies that the marginal productivity in turn depends on the proportions of consumers types in the jurisdiction.[17] Therefore, in our context, it is legitimate to write the wages as a function of the crowding profile of proportions, i.e. $\alpha_\omega^\theta(\beta_\omega), \forall\theta \in \mathbf{\Theta}$.

As in the classic work of Debreu [13] the consumers of the jurisdiction own the resources and control the production. Let $\phi_\omega^\theta \geq 0$ represent a type θ consumer's share of ownership of the private production profits in jurisdiction ω such that $\sum_{\theta\in\mathbf{\Theta}} n_\omega^\theta \phi_\omega^\theta = 1$. Ownership depends on workers' jurisdiction choice, which is similar to labor-managed firms. Observe that in our framework these types of firms are not inefficient since we are considering a CRS production technology.[18]

CONSUMERS

Consumers have well defined preferences on the consumption of private goods $x \in \mathbb{R}_+^{|L|}$ and public project $g \in \mathbf{G}$ and its associated level of congestion $n \in \mathbb{N}_+$, represented by a utility function $\tilde{u}^{\theta_i}(x, g, n)$. The next two assumptions are common in the literature.

Assumption 3: *For every consumer θ^i, the utility function $\tilde{u}^{\theta_i}(x, g, n)$ is continuous, strictly monotonic and strictly quasiconcave in x, decreasing in n for $g \neq \emptyset$, and bounded for all possible $g \in \mathbf{G}$.*

Assumption 4: *For empty public projects, $g = \emptyset$, congestion is assumed to be irrelevant, that is, $\forall n \neq n'$, $\tilde{u}^{\theta_i}(x, \emptyset, n) = \tilde{u}^{\theta_i}(x, \emptyset, n)$.*

The following assumption is similar to Mas-Colell [28] and Wooders [41]. It says that the utility at zero consumption of the commodities is that of choosing the worst allocations of commodities, public project and level of congestion.

[17] Recall that a CRS production function implies that the optimal production level remains constant as long as we keep with the optimal proportions of workers' types in the jurisdiction.

[18] We could have ignored ownership issues without affecting our model and results.

Assumption 5: *Let* $\underline{\mathrm{u}}^{\theta_i} \equiv \min_{(x',g',n')} \tilde{u}^{\theta_i}(x',g',n')$. *Then,* $\tilde{u}^{\theta_i}(0,g,n) = \underline{\mathrm{u}}^{\theta_i}$, $\forall(\theta,g,n) \in \mathbf{\Theta} \times \mathbf{G} \times \mathbb{N}_+$.

All consumers of the same type are endowed with a strictly positive vector of commodities, i.e. $e^\theta \in \mathbb{R}^{|L|}_{++}$, $\forall\theta \in \mathbf{\Theta}$. We also assume that the aggregate endowments are bounded from above, that is, $\exists E \in \mathbb{R}^{|L|}_{+}$ finite such that $0 < \int_{\mathbf{I}} e^i dv < E$. We allow the consumers to trade private goods not only with the members of their own jurisdiction, but also with the consumers of other jurisdictions.

Once a consumer is in a jurisdiction $\omega = (g_\omega, (n^\theta_\omega)_{\theta\in\mathbf{\Theta}}, y_\omega)$ he takes as given the public project g_ω, the profile of consumers $(n^\theta_\omega)_{\theta\in\mathbf{\Theta}}$, the production technology y_ω, but also the tax t^θ_ω he is charged in order to contribute to finance the public good provision (discussed below). We denote the utility of consumer θ_i in a jurisdiction type ω by $u^{\theta_i}\left(x^{\theta_i}_\omega, \omega\right)$. Observe that this is an indirect utility function on the primitive $\tilde{u}^{\theta_i}$: the consumption of private goods $x^{\theta_i}_\omega \in \mathbb{R}^{|L|}_{+}$ and public project g_ω and its associated level of congestion n_ω enter directly into the utility function u^{θ_i}, whereas the lump-sum tax t^θ_ω and production technology $y_\omega((n^\theta_\omega)_{\theta\in\mathbf{\Theta}})$ enter indirectly in u^{θ_i} through the consumer's budget constraint:

$$p(x^{\theta_i}_\omega - e^\theta) + t^\theta_\omega \leq \alpha^\theta_\omega(\beta_\omega) + \phi^\theta_\omega \Pi^y_\omega \qquad ((\mathrm{BC}(\theta_i)))$$

The budget constraint $\mathrm{BC}(\theta_i)$ says that the sum of the cost of the purchased commodities and the lump sum tax payment to the jurisdiction manager must be smaller than or equal to the income obtained from the consumer's endowments, wage (which depends on the crowding profile of proportions) and share of the private production profits. Observe that all consumers of the same type in a jurisdiction are subject to the same lump-sum taxes (since the preferences for public goods and labor skills are observed, but not the preferences for the consumption of private goods), the same wages (since consumers of the same type have the same labor skills, as stated above) and the same share of ownership of the benefits of the private production.

In an economy where the private goods are used as inputs for the production of public projects, we have to prevent that the "minimum expenditure situation" pointed out by Ellickson et al. [15], among others. For that we need to consider an extra assumption, which says that, if the entire social endowment of private goods is used to produce public projects, then, for almost every consumer $\theta_i \in \mathbf{I}$, there exists some good $l \in \mathbf{L}$ and some sufficiently large level of consumption of this good l such that every agent would prefer consuming his endowment together with this large level of good l, and belong to no jurisdiction, rather than to consume the bundle $x_{\omega}^{\theta_i}$ in a jurisdiction ω.

Assumption 6: *Let δ_l be a vector in $\mathbb{R}_+^L$ consisting of one unit of the l^{th} commodity and nothing else. If the entire social endowment of private goods is used to produce public projects, then, for almost every consumer $\theta \in \mathbf{\Theta}$, $\exists l \in L$ and large $r > 0$ such that $u^{\theta_i}(e^{\theta}+r\delta_l, \omega') > u^{\theta_i}(x_{\omega}^{\theta_i}, \omega)$, where $\omega = (g_{\omega}, n_{\omega}, y_{\omega}, \beta_{\omega})$ and $\omega' = (0, 0, y_{\omega}, \beta_{\omega})$.*

The consumer's optimization problem is, first to choose the best consumption bundle given the price level p, the lump sum tax t_{ω}^{θ} and the jurisdiction type $\omega = (g_{\omega}, \left(n_{\omega}^{\theta}\right)_{\theta \in \mathbf{\Theta}}, y_{\omega})$, and then, to choose the jurisdiction type that gives him the highest indirect utility. In the former,

$$U^{\theta_i}\left(\omega, p, t_{\omega}^{\theta}\right) \equiv \max_{\{x_{\omega}\}} u^{\theta_i}\left(x_{\omega}, \omega\right) \qquad \text{(Problem 1)}$$
$$\text{s.t} \quad p(x_{\omega} - e^{\theta}) + t_{\omega}^{\theta} \leq \alpha_{\omega}^{\theta}(\beta_{\omega}) + \phi_{\omega}^{\theta}\Pi_{\omega}^{y} \qquad (\mathrm{BC}(\theta_i))$$

The function $U^{\theta_i}\left(\omega, p, t_{\omega}^{\theta}\right)$ is the maximal utility that consumer θ_i can achieve with the jurisdiction package $(g_{\omega}, (n_{\omega}^{\theta})_{\theta \in \Theta}, y_{\omega})$, given prices p and tax t_{ω}^{θ}. Let us denote the consumer θ_i's demand function[19] for private goods at jurisdiction ω by $b(\theta_i, \omega) \equiv \mathrm{argmax}_{\{x_{\omega}\}}\, u^{\theta_i}\left(x_{\omega}, \omega\right)$ such that $(\mathrm{BC}(\theta_i))$ holds, and let $b : I(\theta) \rightarrow \mathbb{R}_+^{|L|}$ be a measurable function from the continuum of type θ consumers at jurisdiction ω into $\mathbb{R}_+^{|L|}$. The demand function is continuous (see Claim 1 in the Appendix).

Notice that the jurisdiction type implicitly depends on the crowding profile of proportions $\beta_{\omega} = (\beta_{\omega}^{\theta})_{\theta \in \mathbf{\Theta}}$ (through the wages $\alpha_{\omega}^{\theta}(\beta_{\omega})$), and, therefore, we can write the indirect utility function as $U^{\theta_i}(g_{\omega}, n_{\omega}, y_{\omega}, \beta_{\omega}, p, t_{\omega}^{\theta})$. That is, we can write the indirect utility function as describing an economy with *anonymous crowding* in the consumption of public goods (n_{ω} as a direct argument), but also considering U^{θ_i} to depend on the proportions of consumers types, which in turn determine the amount of private goods that the consumer can purchase (through the wages), affecting in this way his utility. This possibility implies that the formation of an heterogeneous community (more than one type of consumer in the jurisdiction) is compatible with anonymous crowding since consumers may obtain high wages in a jurisdiction with collaborative production. In other words, when

[19] Observe that this function is well defined since $u^{\theta_i}\left(x_{\omega}; \omega\right)$ is srictly quasiconcave in x_{ω} (Assumption 3).

choosing a jurisdiction type, the consumers care not only about the jurisdiction public project, but also about the wages earned in the private production sector. This situation occurs in real life, where individuals choose a place (district, city, county) to live given the public projects offered (water supply systems, parks, transport facilities, hospitals, education, etc.) and the contracts offered to their respective labor skills.[20]

After solving Problem 1, consumer θ_i chooses his best jurisdiction type that maximizes his indirect utility $U^{\theta_i}\left(\omega, p, t_\omega^\theta\right)$, that is,

$$\omega(\theta_i) \equiv \arg\max_{\omega \in \mathbf{\Omega}} U^{\theta_i}\left(\omega, p, t_\omega^\theta\right) \qquad \text{(Problem 2)}$$

The solution to Problem 2 gives the consumer θ_i's demand for a membership in jurisdiction ω. Observe that we allow for consumers of the same type to have different preferences on the consumption of private goods, and, therefore, it may occur that consumers of the same type choose a different jurisdiction type. We represent the pure strategy of consumer θ_i by a basis vector $\omega(\theta_i)$ of dimension $|\Omega|$. The vector $\omega(\theta_i)$ is the vector in $\mathbb{R}^{|\Omega|}$ with one as ω^{th} coordinate and zero otherwise.

Notice that, the set of jurisdiction memberships coincides with the set of consumers, since we require that each consumer chooses only one jurisdiction as the place to live. This allows us to define a measure space of jurisdiction memberships $(\mathbf{M}, \mathcal{M}, \upsilon)$ that is homeomorphic to the space of consumers $(I, \mathcal{F}, \upsilon)$, where $\mathbf{M}$ is the set of jurisdiction memberships and $\mathcal{M}$ is a σ-algebra of subsets of $\mathbf{M}$. In fact, this distinction is only introduced in order to distinguish between consumers and memberships. Let us denote by $M(\omega, \theta) \in \mathcal{M}$ the set of jurisdiction ω memberships demanded by type θ consumers, that is,

$$M(\omega, \theta) = \left\{\theta_i : \omega(\theta_i) \in \arg\max_{\omega \in \mathbf{\Omega}} U^{\theta_i}\left(\cdot, p, t_\omega^\theta\right)\right\}$$

Then, the total demand for memberships in a jurisdiction ω is $M(\omega) = \cup_{\theta \in \mathbf{\Theta}} M(\omega, \theta)$. By taking the union over the sets $M(\omega)$ we get $\mathbf{M}$.

[20] As we explain in the introduction, we prevent workers to consumer public goods in one jurisdiction while working elsewhere. This makes our analysis tractable. However, this might not always the case. It would be interesting to explore the possibilities of removing this assumption. See Haughwout and Inman [20] for an inspiring explanation of how suburban citizens benefit from city produced public goods and infrastructure. We leave this extension for future research.

We now impose a measurement condition on the set of demanded jurisdiction memberships, already pointed out by Kaneko and Wooders [21] and Ellickson et al. [15].

The measurement condition (MC):

(MC.i) *The sets* $M(\omega, \theta) \in \mathcal{M}$ *are* υ*-measurable.*

(MC.ii) *There exists a measure* μ_ω *of type* ω *jurisdictions such that* $\upsilon (M(\omega, \theta)) \equiv n^\theta_\omega \times \mu_\omega, \forall \omega, \theta \in (\mathbf{\Omega} \times \mathbf{\Theta})$.

Condition (MC.ii) guarantees that jurisdiction choices are consistent across the population. Observe also that under this condition the proportions that hold in a type ω jurisdiction are maintained once we integrate over the existing jurisdictions of this type, that is, $\beta^\theta_\omega = \frac{\upsilon(M(\omega,\theta))}{\upsilon(M(\omega))} = \frac{n^\theta_\omega}{n_\omega}, \forall(\omega, \theta) \in \mathbf{\Theta} \times \mathbf{\Omega}$. Moreover, the relative proportions of consumers types in a jurisdiction ω are also maintained: $\upsilon (M(\omega, \theta)) = \beta_{\theta,\theta'} \upsilon (M(\omega, \theta'))$, $\forall (\omega, \theta, \theta') \in \mathbf{\Omega} \times \mathbf{\Theta} \times \mathbf{\Theta}$, where $\beta_{\theta,\theta'} = (n^\omega_\theta / n^\omega_{\theta'})$ is the relative proportion of type θ consumers with respect to type θ' consumers.[21]

JURISDICTION MANAGERS

So far we have modeled an economy with a continuum of consumers, represented by the non-atomic measure space $(I, \mathcal{F}, \upsilon)$, and the corresponding (homeomorphic) space of jurisdiction memberships $(M, \mathcal{M}, \upsilon)$. As argued above, Assumptions 1 and 2 imply that the continuum of consumers split into finitely populated jurisdictions, and this implies a continuum of jurisdictions. So let us denote this space of jurisdictions by $(\mathbf{J}, \mathcal{J}, \mu)$, where $\mathbf{J}$ is the set of jurisdictions, $\mathcal{J}$ is a σ-algebra of subsets of $\mathbf{J}$ and μ is a nonatomic finite measure on $\mathcal{J}$. The set of jurisdictions of type $\omega \in \mathbf{\Omega}$ is denoted by $J(\omega) \in \mathcal{J}$, and has an associated measure denoted by $\mu_\omega = \mu(J(\omega))$.

We refer to a jurisdiction of type ω (the policy package offered) by ω (since all of them are identical) and to its associated jurisdiction manager by $j(\omega)$. Observe that our framework of negligible jurisdictions does not speak against a model with non-negligible countries if one considers a spatial context with homogeneous jurisdictions in a region. This is just a matter of aggregating

[21] To see this, note that $n^\theta_\omega = |M(\omega, \theta, s)| = \beta_{\theta,\theta'} \left|M(\omega, \theta', s)\right| = \beta_{\theta,\theta'} n^{\theta'}_\omega$. Then by integrating over all subsets $M(\omega, \theta, s)$, we have that $\upsilon (M(\omega, \theta)) = \beta_{\theta,\theta'} \upsilon \left(M(\omega, \theta')\right)$, as desired.

jurisdictions into countries. To see this, let us integrate over the continuum of type ω jurisdictions, $J(\omega) = \int_{J(\omega)} J(\omega, j) d\mu_{\omega}$. One can think of $J(\omega)$ as an atomic country of type ω. Since the sets of jurisdictions types $\mathbf{\Omega}$ and consumers types $\mathbf{\Theta}$ are finite, it follows that the set of possible countries is finite.

We now indicate how the jurisdiction manager offers the policy package in a competitive way. We start by stating his public good provision problem. The cost function for providing the public project g_{ω} is given by $c(g_{\omega}) \in \mathbb{R}_{+}^{|L|}$, which is assumed to be bounded.[22] In order to highlight the labor complementarities in the private production sector, we assume that the production of the public projects uses only commodities as inputs and not labor inputs.

Next, let us model *smart* jurisdiction managers in the sense that they offer a jurisdiction type ω whenever there are profit opportunities. This condition, originally proposed by Konishi [23] for a Tiebout economy, is referred as Exhausted Profit Opportunities (EPO). As Konishi shows, the EPO condition guarantees efficiency through the entrepreneurship of the jurisdiction managers: the profit opportunities are sought by the jurisdiction managers. This characterization dispenses with the assumption of price completeness required by Ellickson et al. [15], which says that every possible jurisdiction type does have a price regardless of the existence of equilibrium. Here the market becomes complete due to the initial entrepreneurial deviations of the jurisdiction managers.

E.P.O (Konishi [23]) - *Exhausted Profit Opportunities by jurisdiction managers: For all ω with $\sum_{\theta \in \mathbf{\Theta}} t_{\omega}^{\theta} n_{\omega}^{\theta} > pc(g_{\omega})$, we have that $\exists \omega' \in \mathbf{\Omega}$ offered by a jurisdiction manager $j(\omega') \in \mathbf{J}$ such that $\forall \theta_i$,*

$$U^{\theta_i}\left(\omega', \bar{p}, \bar{t}_{\omega'}\right) > \max_{\{x_{\omega}\}} u^{\theta_i}(x_{\omega}, \omega)$$

where $U^{\theta_i}\left(\omega', \bar{p}, \bar{t}_{\omega'}\right)$ is as defined in Problem 1.[23]

In words, the characterization of exhausted profit opportunities by jurisdiction managers says that if there is a jurisdiction type ω with a profit opportunity, then there must appear another jurisdiction manager $j(\omega')$ of type $\omega' \in \mathbf{\Omega}$ that gives a higher indirect utility to all consumer types such that their budget constraints hold. The EPO condition can be seen as a way of modeling the competitive

[22] It may happen that the cost of providing the public goods is zero as it would be the case of the natural resources (e.g. sun, beach).

[23] $U^{\theta_i}\left(\omega', \bar{p}, \bar{t}_{\omega'}\right)$ is defined such that $u^{\theta_i}\left(x_{\omega'}, \omega'\right)$ satisfies the consumer's budget constraint at jurisdiction ω'.

behavior of jurisdiction managers in a way that they adapt their offer to the demand of consumers and such that the competition assures zero profits. Observe that EPO and MC conditions assure the consistency between the supply (inelastic) and demand of jurisdiction memberships.

4 Equilibrium

Definition CE: A *competitive equilibrium* for our economy consists on a vector of consumption of private goods $(\bar{x}(\theta_i))_{\theta_i \in \mathbf{I}}$ and the respective market price $\bar{p}$, a set of jurisdiction memberships $(\bar{M}(\omega, \theta),)_{\omega \in \mathbf{\Omega}, \theta \in \mathbf{\Theta}}$, a vector of lump sump taxes $(\bar{t}^{\theta}_{\omega})_{\theta \in \mathbf{\Theta}, \omega \in \mathbf{\Omega}}$ and wages $(\bar{\alpha}^{\theta}_{\omega})_{\theta \in \mathbf{\Theta}, \omega \in \mathbf{\Omega}}$, and a policy package $\omega = (\bar{g}_{\omega}, (\bar{n}^{\theta}_{\omega})_{\theta \in \mathbf{\Theta}}, \bar{y}_{\omega})$ for each jurisdiction type such that

(CE.1.1) Consumers choose optimally their consumption bundle: if $\exists \tilde{x}(\theta_i)$ such that $u^{\theta_i}(\tilde{x}(\theta_i), \omega) > u^{\theta_i}(\bar{x}(\theta_i), \omega))$, then $\bar{p}(\tilde{x}(\theta_i) - e^{\theta}) + \bar{t}^{\theta}_{\omega} > \bar{\alpha}^{\theta}_{\omega}\left(\bar{\beta}_{\omega}\right) + \phi^{\theta}_{\omega}\bar{\Pi}^{y}_{\omega}$.

(CE.1.2) Consumers choose optimally their jurisdiction membership $\bar{\omega}(\theta_i)$, which in turn determine $(\bar{M}(\omega, \theta),)_{\omega \in \mathbf{\Omega}, \theta \in \mathbf{\Theta}})$ and the associated consumption bundle $\bar{x}(\theta_i, \omega(\theta_i))$;

(CE.2) The competition among the jurisdiction managers exhausts profits, i.e. $\sum_{\theta \in \mathbf{\Theta}} \bar{t}^{\theta}_{\omega}\bar{n}^{\theta}_{\omega} = \bar{p}c(\bar{g}_{\omega}), \forall \omega \in \mathbf{\Omega}$;

(CE.3) Wages are paid according to the consumers' marginal contribution to the private production process such that $\Pi^{y}_{\omega} = 0$;

(CE.4) The private goods market clears, i.e.

$$\sum_{\theta \in \mathbf{\Theta}} \int_{I(\theta)} \left(\bar{x}(\theta_i) - e^{\theta_i}\right) d\upsilon + \sum_{\omega \in \mathbf{\Omega}} \bar{\mu}_{\omega}(c\left(\bar{g}_{\omega}\right) - \bar{y}_{\omega}(\left(\bar{n}^{\theta}_{\omega}\right)_{\theta \in \mathbf{\Theta}})) = 0$$

Remark 1 (MC): Observe that in equilibrium the MC condition is such that it assures the existence of a measure μ_{ω} (supply of type ω jurisdictions) such that $\upsilon\left(\bar{M}(\omega, \theta)\right) \equiv \bar{n}^{\theta}_{\omega} \times \mu_{\omega}$, $\forall \theta \in \mathbf{\Theta}$, with $\bar{n}^{\theta}_{\omega} \in \mathbb{N}_{+}$. That is, the supply of jurisdictions adapts in equilibrium to the demand by the consumers such that consistency across the population holds.

Theorem 1: *There exists an competitive equilibrium in pure strategies for our local public good economy with collaborative private production and anonymous crowding in the consumption of public goods.*

The proof of existence of equilibrium for this Berglas' economy could be drawn by following Ellickson et al. [15] or Conley and Wooders [11] if one carefully elaborates on these models to integrate a CRS private production technology depending on the crowding profile, and the assumption that there is a proper distribution between tastes and labor skills.[24] The technique we use here to show existence of a competitive equilibrium differs from both Conley and Wooders [11] and Ellickson et al. [15]. Wooders [41], Allouch and Wooders [2], among others, first prove that equilibrium exists by showing that the core is non-empty. Then Conley and Wooders' [11] decentralization result between the core and the set of competitive equilibrium for an economy with crowding types may apply. Ellickson et al. [15] prove existence of a competitive equilibrium for a decentralized price-taking economy by using the "no excess demand" approach. Our approach to prove existence of equilibrium is by "simultaneous optimization".[25] That is, we investigate the problem of existence of a competitive equilibrium by transforming it into a problem of existence of a social system equilibrium (in terms of Arrow and Debreu [4]), where the agents simultaneously seek to maximize their respective payoff functions. As Arrow and Debreu [4] assert, we are able to test in a clearer way the consistency of the equations that describe the model.

In the "simultaneous optimization approach" each player maximizes a payoff function on a constraint set. Both the payoff function and the constraint set may be parameterized by the other players' actions. This second dependence does not occur in games. The extension is a mathematical object referred to as a "generalized game" by Debreu [14].

Next we sketch the important steps in the proof of existence of a pure strategies competitive equilibrium.

1. We extent the generalized game to mixed strategies where each consumer chooses a mixed strategy on $\mathbf{\Omega}$ in their Problem 2. By Debreu's [14] theorem we can assert that the extended generalized game has an equilibrium in mixed strategies.

2. By noticing that the Price auctioneer's objective function in the extended

[24] One may be tempted to modify Ellickson et al. [15] model in order to accommodate private production as follows. Allow people to join exactly two clubs, one a standard jurisdiction and the second a club consisting of only themselves. The second club would produce private goods. However, we think that this approach is not satisfactory since then we would be loosing the trade-off (public versus private sectors) occuring in just a single jurisdiction (or club).

[25] See the details of both approaches in Debreu ([12]).

game depends on the consumers' mixed strategies only through finitely many indicators, one for each type $\theta \in \mathbf{\Theta}$, we can guarantee that the equilibrium is in fact a pure strategies equilibrium (by Araujo and Páscoa [3, Lemma 2] and Páscoa [31]). It is important to notice that Schmeidler's [35] purification result cannot be applied in our setting, since consumers with the same type may have different preferences on private goods, that is, there is one common best response $b(\theta_i, \omega)$ for each consumer of type θ. Schmeidler's result could only be applied if instead we had a common best response $b(\theta, \omega)$ for type θ consumers.

3. We show that the equilibrium of the generalized game is in fact a competitive equilibrium.

Perhaps as important as the issue of existence of competitive equilibria are the problems of normative or welfare economics. In the rest of this section we address the question of whether the allocation of resources in a competitive equilibrium is efficient in the sense of Pareto. As we pointed out above, efficiency is attained through the competition among jurisdiction managers and the freedom of the consumers to "vote with their feet" for their most preferred jurisdiction. The entrepreneurship of the jurisdiction managers assures that the demanded jurisdiction types are provided whenever they are profitable. In equilibrium the profits are exhausted by the competition among jurisdiction managers.

Let us consider the set $\mathbf{\Omega}^*$ of jurisdiction types that are offered by the jurisdiction managers to the consumers. We are in a context where any deviation of consumers to a new desired feasible jurisdiction type is facilitated by the jurisdiction managers in the form of a new jurisdiction. Let the outcome $\bar{\kappa} = (\bar{x}(\theta_i), u^{\theta_i}(\bar{x}(\theta_i), \bar{\omega}))_{\theta_i \in \mathbf{I}}$ with $\bar{\omega} \in \mathbf{\Omega}^*$, where consumers' budget constraints (BC) are satisfied (feasibility). We say that a measurable subset $I \subset \mathcal{I}$ of the total population of consumers *improves upon* $\bar{\kappa}$ with a feasible outcome, say $\tilde{\kappa} = (\tilde{x}(\theta_i), u^{\theta_i}(\tilde{x}(\theta_i), \tilde{\omega}))_{\theta_i \in \mathbf{I}}$ with $\tilde{\omega} \in \mathbf{\Omega}^*$, if for every $\theta_i \in \mathbf{I}$, $u^{\theta_i}(\tilde{x}(\theta_i), \tilde{\omega}) \geq u^{\theta_i}(\bar{x}(\theta_i), \bar{\omega})$ and $u^{\theta_i}(\tilde{x}(\theta_i), \tilde{\omega}) > u^{\theta_i}(\bar{x}(\theta_i), \bar{\omega})$ for at least one $\theta_i \in \mathbf{I}$.[26]

A feasible outcome is *efficient* in the *Pareto* sense if there is no measurable subsets of consumers that can improve upon it.

[26]Consistent with Assumption 2 we require that the improving coalitions be finite (Kaneko and Wooders [21]). The *f-core*, or simply the *core*, of the economy consists of those feasible states of the economy $(\bar{x}(\theta_i, \bar{\omega}), u^{\theta_i}(\bar{x}(\theta_i), \bar{\omega}))_{\theta \in \Theta}$, $\bar{\omega} \in \mathbf{\Omega}^*$, with the property that, for some subset of consumers $I^0 \subset \mathbf{I}$ of full measure, there is no finite coalition $\hat{I} \subset I^0$ that can improve upon $(\bar{x}(\theta_i, \bar{\omega}(\theta_i)), u^{\theta}(\bar{\omega}(\theta_i, \bar{\omega})))_{\theta \in \Theta}$.

Theorem 2. *The resulting competitive equilibrium is Pareto efficient.*

Finally, we focus our attention on the conditions that lead to existence of an heterogeneous community. Let us consider the following *index of heterogeneity* (H) in a jurisdiction, which compares the proportions over pairs of consumers' types:

$$H_\omega(\theta,\theta') = (\max_{\theta} \bar{\beta}^{\theta}_{\omega} - \min_{\theta'} \bar{\beta}^{\theta'}_{\omega}) \in [0,1]$$

where $\beta^{\theta}_{\omega} = \upsilon\left(M^{\theta}_{\omega}\right) / \sum_{\theta\in\Theta} \upsilon\left(M^{\theta}_{\omega}\right)$ refers to the proportion of type θ memberships in a jurisdiction ω. We say that a community is mixed (or heterogeneous) when there is at least more than one consumer type in it, that is, $\exists(\theta,\theta') : H_\omega(\theta,\theta') \neq 1$ and $\min\{\bar{\beta}^{\theta}_{\omega}, \bar{\beta}^{\theta'}_{\omega}\} \neq 0$. We denote the crowding profile of proportions of a heterogeneous community by β_{HET}. When $H_\omega(\theta,\theta') = 0$ and $n^{\theta}_{\omega} > 0$ the two types θ and θ' of consumers are in the same proportions in a jurisdiction type ω, and therefore, the jurisdiction type ω is *uniformly heterogeneous* on these two types of consumers. When $H_\omega(\theta,\theta') > 0$ and $\min\{\bar{\beta}^{\theta}_{\omega}, \bar{\beta}^{\theta'}_{\omega}\} \neq 0$, the jurisdiction ω is mix-populated by the two types of consumers but in different proportions. If $H_\omega(\theta,\theta') = 1, \forall\theta'$, then the jurisdiction type has an *homogeneous* population of type θ consumers. We denote the crowding profile of proportions of a homogeneous community by $\beta_{HOM(\theta)}$, which has a vector $(0, ..., 0, 1, 0, ..., 0)$ with 1 in the θ component but 0 otherwise.

It is easy to see that, in a context of *anonymous crowding* in the consumption of public goods, there are two *necessary conditions* for the existence of a mixed community: a collaborative production function and a correlation between tastes and labor types. Both of them are considered in our framework. In the absence of a collaborative production process the well known result that homogenous groups coalesce around like individuals would apply in an economy with anonymous crowding (see Scotchmer and Wooders [36] for this result and McGuire [29] for a detailed normative analysis on this issue).

A sufficient condition for a heterogeneous community to result in equilibrium is that, for every agent θ_i and every possible configuration $\tilde{\beta}_{HOM(\theta)}$, the following holds:

$$U^{\theta_i}(\bar{\omega}_{HET}; \bar{\beta}_{HET}) > U^{\theta_i}(\tilde{\omega}_{HOM}; \tilde{\beta}_{HOM(\theta)}), \;\; \forall\theta \in \mathbf{\Theta} \tag{SC1}$$

where $\bar{\omega}_{HET} \in \arg\max U^{\theta_i}(\cdot; \bar{\beta}_{HET})$ and $\tilde{\omega}_{HOM(\theta)} \in \arg\max U^{\theta_i}(\cdot; \tilde{\beta}_{HOM(\theta)})$.

A more intuitive condition could be obtained if one considers a separable utility function. However, Assumption 6, which guarantees that private goods are *essensial* (Mas-Colell [27]), is restrictive in that it excludes separability between private and local public goods. Under certain assumptions one can dispense of this assumption[27], and the following separable utility function could be considered:

$$u^{\theta_i}(x_\omega, \omega) \equiv u^{\theta_i}(x_\omega, y_\omega, \beta_\omega, p) + u^{\theta_i}(g_\omega, n_\omega, t^\theta_\omega)$$

The sufficient condition would become: $\forall\theta_i, \forall\theta \in \boldsymbol{\Theta}$ and every possible $\tilde{\beta}_{HOM(\theta)}$,

$$u^{\theta_i}(\bar{x}_{\bar{\omega}_{HET}}, \bar{y}_{\bar{\omega}_{HET}}, \bar{\beta}_{HET}, \bar{p}) + u^{\theta_i}(\bar{g}_{\bar{\omega}_{HET}}, \bar{n}_{\bar{\omega}_{HET}}, \bar{t}^\theta_{\bar{\omega}_{HET}}) > U^{\theta_i}(\tilde{\omega}_{HOM(\theta)}; \tilde{\beta}_{HOM(\theta)}) \tag{SC2}$$

where $\bar{x}_{\bar{\omega}_{HET}}$ is a solution to problem 1 at jurisdiction $\bar{\omega}_{HET}$. If (SC2) happens, it may occur that for some consumers' types the public project $\bar{g}_{\bar{\omega}_{HET}}$ offered in the mixed jurisdiction $\bar{\omega}_{HET}$ is less preferred than the public project $\tilde{g}_{\tilde{\omega}_{HOM(\theta)}}$ that would have been chosen if the jurisdiction is populated only by the same type consumers, i.e. $u^{\theta_i}(\bar{g}_{\bar{\omega}_{HET}}, \bar{n}_{\bar{\omega}_{HET}}, \bar{t}^\theta_{\bar{\omega}_{HET}}) < u^{\theta_i}(\tilde{g}_{\tilde{\omega}_{HOM(\theta)}}, \tilde{n}_{\tilde{\omega}_{HOM(\theta)}}, \tilde{t}^\theta_{\tilde{\omega}_{HOM(\theta)}})$. However, (SC2) guarantees that the loss in utility is more than offset by the higher wages that the consumer obtains in the heterogeneous community, which allow her to increase her consumption of private goods and hence her utility.

5 Final Remarks

This paper addressed the group composition problem that consumers face when the production in the jurisdiction is collaborative and there is a correlation between the distribution of tastes and labor skills. For this economy, we showed that a competitive equilibrium exists and that it is efficient.

Several possibilities are open for future investigation. An alternative modeling of the competitive behaviour of the jurisdiction managers would be to specify their payoff functions and budget constraints (e.g. modelling bureaucratic behavior as in Shleifer and Vishny [38]). However, one should take care of assuring the consistency between the supply and demand of jurisdiction memberships. However, this task seems not easy, since only a unique measure of type ω jurisdictions (μ_ω) should accommodate the supply and demand of jurisdiction ω memberships for every consumer type. This suggests that the equilibrium would result in a

[27] Guilles and Scotchmer [17] show that the assumption that private goods are "essential" is not required if one considers the conditions of "exhaustion of blocking opportunities" and "efficient scale".

high degree of indeterminacy of μ_ω. Recall that in our framework consistency holds since supply is assumed to accommodate demand (see also Konishi [22] and [23]).

Another open question would be to consider the tastes for public goods and labor skills as unobservable characteristics, as in Conley and Wooders [11] and Zame [42], respectively. Then one may wonder how these information problems can affect the composition of the communities.

6 Appendix

We decompose the Appendix in two parts. In Part A we construct the generalized game and indicate some useful results that will help for the proofs of the Theorems below. Appendix B is devoted for the proofs of Theorems 1 and 2.

6.1 Appendix A

Claim 1: *The demand function $b(\theta_i, \omega)$ has nonempty compact values and is continuous.*

Proof. In the consumers' maximization Problem 1 each consumer θ_i maximizes $u^{\theta_i}(x;\omega)$ on his budget set BC(θ_i). Let us denote consumer θ_i's budget constraint correspondence by $B(\theta_i, p, \omega) = \{x_\omega^{\theta_i} \in \mathbf{X} : BC(\theta_i) \text{ holds}\}$, which is continuous. Here $\mathbf{X}$ denotes the strategy set, which is non-empty, convex and compact. Compactness follows because total endowments are finite, i.e. $x \in \left[0, \int_{\mathbf{I}} e^i dv\right]$. By strictly quasiconcavity of u^{θ_i} and convexity of the values of $B^{\theta_i}(p,\omega)$, it follows that the budget constraint correspondence has convex values. Consumers' utilities $u^{\theta_i}(x_\omega, \omega)$ are continuous, strictly quasiconcave and strictly monotonic in x from Assumption 3. Interiority of endowments guarantees the positivity of consumers' incomes and this suffices to establish the lower semi-continuity of the budget constraints (see Debreu [13]). Now, by Berge's Maximum theorem, the demand function $b(\theta_i, \omega)$ is continuous. ■

THE GENERALIZED GAME:

Here our objective is to assure the existence of a solution, in the sense that the equations that describe the model are consistent with each other. This is a fundamental step that has to be done before aiming at any empirical test of the model. Let us consider the economy introduced in Section 3 satisfying Assumptions 1 to 6, EPO and MC. In the generalized game an agent k chooses his strategy X_k parametrized by the other agents' strategies $\bar{X}_{-k}$. The generalized game for this economy is played by the consumers, a Price auctioneer, a Wage auctioneer and two tax auctioneers. The **Price auctioneer** chooses a vector of commodity prices p, given the consumers' demands $(\bar{x}_{\omega}^{\theta_i})_{\theta_i \in \mathbf{I}}$, and the policy package offered by each jurisdiction type, in order to maximize the following payoff function:

$$p(\sum_{\theta\in\mathbf{\Theta}} \int\limits_{I(\theta)} \left(\bar{x}(\theta_i) - e^{\theta_i}\right) dv + \sum_{\omega\in\mathbf{\Omega}} \bar{\mu}_{\omega}(c\left(\bar{g}_{\omega}\right) - \bar{y}_{\omega}(\left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}}))) \qquad \text{(Problem 1)}$$

The **Wage auctioneer** chooses a vector of wages $\alpha = (\alpha_{\omega}^{\theta})_{\theta\in\mathbf{\Theta},\boldsymbol{\omega}\in\mathbf{\Omega}}$, given a policy package for each jurisdiction type (in particular, given $(\bar{y}_{\omega}, \left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}})_{\omega\in\mathbf{\Omega}})$ and prices $\bar{p}$, in order to minimize the continuous function

$$\sum_{\omega\in\mathbf{\Omega}}\sum_{\theta\in\mathbf{\Theta}}(\alpha_{\omega}^{\theta} - \frac{\partial y_{\omega}}{\partial m_{\omega}^{\theta}}(\left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}}))^2 \qquad \text{(Problem 4)}$$

where $\frac{\partial y_{\omega}}{\partial m_{\omega}^{\theta}}(\left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}})$ denotes the marginal productivity of a type θ consumer evaluated at the crowding profile $\left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}}$. Recall that the marginal productivity is homogeneous of degree zero in $\left(\bar{n}_{\omega}^{\theta}\right)_{\theta\in\mathbf{\Theta}}$, that is, $\frac{\partial y_{\omega}}{\partial m_{\omega}^{\theta}}$ depends on the crowding vector of proportions.

The EPO condition assures that competition among the jurisdiction managers exhausts profits. In our generalized game we model this by introducing two auctioneers. The first, referred as the **Poll tax auctioneer**, chooses a vector of lump sum poll taxes such that it minimizes the profits of providing the public good in a jurisdiction, i.e. he chooses $\left\{(\tau_{\omega})_{\omega\in\mathbf{\Omega}} : \tau_{\omega} = \tau_{\omega}^{\theta}, \forall\theta\in\mathbf{\Theta}\right\}$, given prices $\bar{p}$ and public projects $(\bar{g}_{\omega})_{\omega\in\mathbf{\Omega}}$, in order to minimize

$$\sum_{\omega\in\mathbf{\Omega}}(\tau_{\omega}\bar{n}_{\omega} - \bar{p}c(\bar{g}_{\omega}))^2 \qquad \text{(Problem 5)}$$

The second, referred as the **Transfers auctioneer**, chooses a vector of transfers among the individuals of a jurisdiction, such that the externalities associated to jurisdiction memberships are internalized[28]. For that,[29] we might consider a system of lump sum transfers $\hat{t}_{\omega_j} = (\hat{t}^{\theta_i}_{\omega_j})_{\theta_i \in M(\theta,\omega_j),\theta\in\mathbf{\Theta}} \in \mathbf{Trans}\,(\omega_j)$, where

$$\mathbf{Trans}\,(\omega_j) \equiv \{\hat{t}_{\omega_j} : \sum_{\theta\in\mathbf{\Theta}} \sum_{\{i:\theta_i\in M(\theta,\omega_j)\}} \hat{t}^{\theta_i}_{\omega_j} = 0\}$$

Observe that we must have $\hat{t}^{\theta_i}_{\omega_j} = \hat{t}^{\theta}_{\omega}$, $\forall\theta_i \in M(\theta,\omega_j)$. Then, we can write the lump-sum tax t^{θ}_{ω} charged on a type θ consumer as $t^{\theta}_{\omega} = \hat{t}^{\theta}_{\omega} + \tau_{\omega}$ (the sum of a lump-sum transfer $\hat{t}^{\theta}_{\omega} \in \mathbf{Trans}\,(\omega)$ and a poll tax $\tau_{\omega} = \frac{pc(g_{\omega})}{n_{\omega}}$). The Transfers auctioneer's problem is to choose $\left\{\hat{t} = (\hat{t}^{\theta}_{\omega})_{\theta\in\mathbf{\Theta},\boldsymbol{\omega}\in\mathbf{\Omega}} : \ \hat{t}_{\omega} \in \mathbf{Trans}\,(\omega)\,, \forall\omega\right\}$, given prices $\bar{p}$ and public projects $(\bar{g}_{\omega})_{\omega\in\mathbf{\Omega}}$, in order to minimize

$$\sum_{\omega\in\mathbf{\Omega}}(\sum_{\theta\in\mathbf{\Theta}}\int_{\bar{M}(\theta,\omega)} \hat{t}^{\theta}_{\omega} dv)^2 \qquad \text{(Problem 6)}$$

given the set of demanded jurisdiction memberships, $\bar{M}(\theta,\omega)_{\theta\in\mathbf{\Theta}}$, and the consumers' budget constraints.

Claim 2: *The Poll tax auctioneer and Transfers auctioneer' strategy sets are non-empty, convex and compact.*

Proof. The strategy sets are non-empty and convex since $t^{\theta}_{\omega} \in \mathbb{R}$. The crowding lump-sum transfers $(\hat{t}^{\theta}_{\omega})_{\theta\in\Theta}$ must be bounded, i.e. $\hat{t}^{\theta}_{\omega} \in [T^-, T^+]$. The upper bound T^+ follows by the assumption that consumer's income is uniformly bounded by $p\int e^i dv + \int \max_{\omega}(\alpha^i_{\omega})dv$, for $p \in \Delta^{L-1}$. The last term $\int \alpha^i_{\omega} dv$ is also bounded since the production is bounded by the upper bound on the jurisdiction population imposed by Assumption 2. That is, we can write $y_{\omega} \leq A$. This in turn implies that $\alpha^{\theta}_{\omega} \leq A$, which guarantees the existence of an upper bound on $\int \alpha^i_{\omega} dv$. The lower bound T^- on the lump sum transfers exists by the argument that if some consumers are paying large negative lump-sum transfers, then others must be paying large positive lump-sum transfers $\hat{t}_{\omega} \in \mathbf{Trans}\,(\omega)$, which implies that some transfers are canceled with some others: $\sum_{\theta\in\Theta} \hat{t}^{\theta}_{\omega} n^{\theta}_{\omega} = 0$. Observe that this argument implies that the lump-sum taxes are bounded, since t^{θ}_{ω} can be written as $\hat{t}^{\theta}_{\omega} + \frac{pc(\bar{g}_{\omega})}{\bar{n}_{\omega}}$, where $\frac{pc(\bar{g}_{\omega})}{\bar{n}_{\omega}}$ is a finite constant component (poll tax), and $c\,(\bar{g}_{\omega}) \geq 0$ is bounded from above by assumption. ■

[28] Observe that different consumers types may have different preferences for a policy package, and thus we have to adapt our model to the well known Samuelson's [34] crowding admission prices.

[29] We follow Conley and Wooders [11] and Ellickson et al. [15], for the notion of a transfers equilibrium, which is necessary in order to obtain compactness of the set of crowding admission lump sum taxes and also to achieve a decentralization result.

Finally, each **consumer** $\theta_i \in \mathbf{I}$ chooses $x(\theta_i, \omega(\theta_i))$, given prices $\bar{p}$, the policy package $\omega = (\bar{g}_\omega, (\bar{n}_\omega^\theta)_{\theta\in\Theta}, \bar{y}_\omega)$, wage $\bar{\alpha}_\omega^\theta$ and tax $\bar{t}_\omega^\theta = \hat{t}_\omega^\theta + \bar{\tau}_\omega$, $\forall \omega \in \mathbf{\Omega}$.

Definition GGE: An equilibrium outcome for the constructed generalized game is a vector $\left(\bar{x}(\theta_i), \bar{M}(\omega, \theta), \bar{g}_\omega, \bar{n}_\omega^\theta, \bar{y}_\omega, \bar{t}_\omega^\theta, \bar{\alpha}_\omega^\theta, \bar{p}\right)_{\theta_i \in I(\theta), \theta\in\mathbf{\Theta}, \omega\in\mathbf{\Omega}}$, such that, for each player the respective action solves his maximization problem parameterized by the other players' actions. That is,

GG.1) Every consumer θ_i chooses optimally $\bar{x}(\theta_i)$ from Problem 1 and $\bar{\omega}(\theta_i)$ from Problem 2 given the price vector $(\bar{p}, \bar{\alpha}_\omega^\theta)$. This determines the demand of jurisdiction memberships by the consumers, $\bar{M}(\omega, \theta)_{\omega\in\mathbf{\Omega}, \theta\in\mathbf{\Theta}}$.

GG.2) The Poll tax auctioneer and Transfers auctioneer choose optimally $((\hat{t}_\omega^\theta)_{\theta\in\mathbf{\Theta}})_{\omega\in\mathbf{\Omega}}$ and $(\bar{\tau}_\omega)_{\omega\in\mathbf{\Omega}}$ from Problems 5 and 6, respectively, given the policy package $(\bar{g}_\omega, (\bar{n}_\omega^\theta)_{\theta\in\mathbf{\Theta}}, \bar{y}_\omega)$ and the prices $\left(\bar{p}, (\bar{\alpha}_\omega)_{\omega\in\mathbf{\Omega}}\right)$. Then, $\bar{t}_\omega^\theta = \hat{t}_\omega^\theta + \bar{\tau}_\omega$, $\forall \theta \in \mathbf{\Theta}, \omega \in \mathbf{\Omega}$.

GG.3) The Price auctioneer chooses a price vector $\bar{p}$ that maximizes the aggregate consumers' budget constraints (Problem 1), given $(\bar{x}(\theta_i))_{\theta_i\in\mathbf{I}}$ and $(\bar{g}_\omega, \bar{y}_\omega(\bar{n}_\omega^\theta)_{\theta\in\mathbf{\Theta}})_{\omega\in\mathbf{\Omega}}$.

(GG.4) The Wage auctioneer chooses optimally $\bar{\alpha} = (\bar{\alpha}_\omega^\theta)_{\theta\in\mathbf{\Theta}, \omega\in\mathbf{\Omega}}$ in Problem 4, given $(\bar{y}_\omega, (\bar{n}_\omega^\theta)_{\theta\in\mathbf{\Theta}})_{\omega\in\mathbf{\Omega}}$ and $\bar{p}$.

6.2 Appendix B

Proof Theorem 1:

We decompose the proof of Theorem 1 in the following two propositions.

Proposition 1: *There exists an equilibrium in pure strategies for the constructed generalized game.*

Proof. Note that the consumers' strategy of choosing their most preferred jurisdiction type in Problem 2 has a finite and discrete space domain $\mathbf{\Omega} = \{1, ..., \omega, ..., \Omega\}$, since the set $\mathbf{\Theta}$ of consumer types, the set $\mathbf{G}$ of public projects, the set $\mathbf{N} \subset \mathbb{N}_+^{|\Theta|}$ of crowding profiles, and the set $\mathbf{Y}$ of production technologies are finite and discrete. In order to circumvent this problem we extent our generalized game to allow for consumers' mixed strategies in the set of jurisdiction types. Then we check that all the conditions of Debreu's [14] theorem hold: action sets are non-empty, convex and compact, the payoff functions are quasiconcave on their own action and continuous on the action space, and the constraint correspondences have non-empty, convex and compact values and are continuous. Then, we purify the equilibrium.

Let us extent consumers' problems to mixed strategies. In order to model this possibility, denote $\Lambda(\mathbf{\Omega}) = \left\{\lambda = (\lambda(\omega))_{\omega\in\mathbf{\Omega}} : \lambda(\omega) \geq 0, \sum_{\omega\in\mathbf{\Omega}} \lambda(\omega) = 1\right\}$. Then $\Lambda(\mathbf{\Omega})$ stands for the convex hull of $(1, ..., \omega, ..., \Omega)$, which is the set of mixed strategies for each consumer. A profile of strategies $\rho : \mathbf{I} \to \Lambda(\mathbf{\Omega})$ brings the continuum of consumers into strategies (pure or mixed).

The consumer θ_i's Problem 1 extended to mixed strategies is such that this consumer randomizes over the possible consumptions in the different jurisdiction types. We write $U^{\theta_i}(\lambda, p, t) \equiv u^{\theta_i}\left(\sum_\omega \lambda(\omega)x(\theta_i, \omega), \lambda, t\right)$, where $x(\theta_i, \omega) = \text{argmax}_{\{x_\omega\}}\ u^{\theta_i}(x_\omega, \omega)$ such that $(\text{BC}(\theta_i))$ holds. That is, consumer randomizes in $\mathbf{\Omega} = \{1, ..., \Omega\}$, but not directly in consumption bundles. Then, consumer θ_i's problem is

$$\max_{\lambda\in\Lambda(\mathbf{\Omega})} \{U^{\theta_i}(\lambda, p, t)\} \qquad \text{(Problem 2}')$$

The utility $u^{\theta_i}\left(\sum_\omega \lambda(\omega)x(\theta_i, \omega), \lambda\right)$ is a continuous bounded real valued function on $\sum_\omega \lambda(\omega)x(\theta_i, \omega)$, and the mixed strategy λ belongs to the convex compact set $\Lambda(\mathbf{\Omega})$. We denote $R(\theta_i) = \{\lambda \in \Lambda(\mathbf{\Omega}) : \lambda \in \arg\max U^{\theta_i}(\lambda, p)\}$ the set of mixed strategies that solve consumer θ_i's Problem $2'$.

Now observe that the prices of private goods are non-negative, and therefore, we can normalize them such that $p \in \triangle^L \equiv \left\{z \in \mathbb{R}^{|L|} : \sum_{k=1}^{L} z_k = 1\right\}$.

We must extent the *fictitious* price auctioneer's problem to allow for consumers' mixed strategies. Given a mixed strategy profile $\rho : \mathbf{I} \to \Lambda(\mathbf{\Omega})$, the vector of consumers' demand for commodities $(\bar{x}(\theta_i, \omega(\theta_i))_{\theta_i\in\mathbf{I}}$, and policy packages $(\omega)_{\omega\in\mathbf{\Omega}}$, the Price auctioneer maximizes the continuous linear function[30]

$$p \to p\left(\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \int_{\mathbf{\Omega}} \left(\bar{x}(\theta_i, \omega(\theta_i)) - e^{\theta_i}\right) d\rho(\theta_i) dv + \sum_{\omega\in\mathbf{\Omega}} \mu_\omega(c\left(\bar{g}_\omega\right) - \bar{y}_\omega\left(\left(\bar{n}_\omega^\theta\right)_{\theta\in\mathbf{\Theta}}\right))\right) \qquad \text{(Problem 3}')$$

on the simplex $\triangle^L$, which implies that the price auctioneer's strategy set is non-empty, convex and compact.

For the Wage auctioneer's problem, first observe that the wage α_ω^θ is bounded

[30]Observe that for the Price auctioneer's objective function we could have written $\sum_{\omega\in\mathbf{\Omega}} \bar{x}(\theta_i, \omega(\theta_i)) d\rho(\theta_i)(\omega)$ instead of $\int_{\mathbf{\Omega}} \bar{x}(\theta_i, \omega(\theta_i)) d\rho(\theta_i)$.

since the marginal productivity is bounded from above, and therefore, the Wage auctioneer's strategies are bounded. The Wage auctioneer's problem is to choose a vector of wages $\alpha = (\alpha_\omega^\theta)_{\theta\in\boldsymbol{\Theta},\boldsymbol{\omega}\in\boldsymbol{\Omega}}$, given prices $\bar{p}$ and a policy package for each jurisdiction type, ($\bar{y}_\omega$ and $\left(\bar{n}_\omega^\theta\right)_{\theta\in\boldsymbol{\Theta}})_{\omega\in\boldsymbol{\Omega}}$, in order to minimize the continuous function

$$\alpha \to \sum_{\omega\in\boldsymbol{\Omega}}\sum_{\theta\in\boldsymbol{\Theta}}(\alpha_\omega^\theta - \frac{\partial y_\omega}{\partial m_\omega^\theta}\left(\left(\bar{n}_\omega^\theta\right)_{\theta\in\boldsymbol{\Theta}}\right))^2 \qquad \text{(Problem 4)}$$

For the Poll tax and Transfers auctioneers' problems 5 and 6 (respectively), observe that if we allow for consumers' mixed strategies, then the tax that each consumer pays in the jurisdiction ω_j is weighted by the probability that type θ consumers assign to that jurisdiction. The associated measure in mixed strategies to $\upsilon(M(\theta,\omega))$ is $\int_{I(\theta)}\rho(\theta_i)(\omega)d\upsilon$. Given a mixed strategy profile $\rho : \mathbf{I} \to \Lambda(\boldsymbol{\Omega})$, the commodity prices $\bar{p}$ and the public project $\bar{g}_\omega$ associated to each jurisdiction type:

- the Poll tax auctioneer chooses a vector $\tau = \tau_\omega \in \mathbb{R}^{|\Omega|}$ that minimizes the continuous function

$$\tau \to \sum_{\omega\in\boldsymbol{\Omega}}(\tau_\omega \sum_{\theta\in\boldsymbol{\Theta}}\int_{I(\theta)}\rho(\theta_i)(\omega)d\upsilon - \mu_\omega\bar{p}c(\bar{g}_\omega))^2 \qquad \text{(Problem 5}')$$

- the Transfers auctioneer chooses a vector $\hat{t} = ((\hat{t}_\omega^\theta)_{\theta\in\boldsymbol{\Theta}})_{\omega\in\boldsymbol{\Omega}} \in \mathbb{R}^{|\Theta||\Omega|}$ that minimizes the continuous function

$$\hat{t} \to \sum_{\omega\in\boldsymbol{\Omega}}(\sum_{\theta\in\boldsymbol{\Theta}}\hat{t}_\omega^\theta\int_{I(\theta)}\rho(\theta_i)(\omega)d\upsilon)^2 \qquad \text{(Problem 6}')$$

given the consumers' demand of jurisdiction memberships (in mixed strategies) and consumers' budget constraints.

Observe that we need the auctioneer's strategy set to be compact. We can assure this because lump-sum tax t_ω^θ charged on a type θ consumer is the sum of a lump-sum transfer $\hat{t}_\omega^\theta$ that belongs to the compact set $\mathbf{Trans}\,(\omega) \equiv \{\hat{t}_\omega : \sum_{\theta\in\boldsymbol{\Theta}}\sum_{\theta_i\in M(\theta,\omega_j)}\hat{t}_\omega^\theta\rho(\theta_i,\omega_j) = 0\}$ and a poll tax $\frac{pc(\bar{g}_\omega)}{\bar{n}_\omega}$. See Claim 2 above for a proof of this argument. Moreover, the Tax auctioneer's strategy space is also non-empty and convex.

By applying Debreu's [14] theorem, we can assert that the extended generalized game has an equilibrium, possibly in mixed strategies. At this point of the proof it remains to show that a degenerate equilibrium of the extended generalized game is in fact an equilibrium of the original game. Recall that from

Assumption 1 there is a continuum of consumers of each type with positive measure. Exact purification of the equilibrium strategies follows the lines of Páscoa [31] and Araujo and Páscoa [3, Lemma 2] purification result. Our framework is even simpler, since purification has to be done within each type $\theta \in \mathbf{\Theta}$ of consumer.

Observe that the Tax auctioneers' objective function depend only on the average of the consumers profile, which satisfies Schmeidler [35] hypothesis, and therefore, also Páscoa [31] and Araujo and Pascoa [3, Lemma 2].

Now observe that the conditions in Páscoa [31] and Araujo and Páscoa' [3, Lemma 2] are satisfied since the Price auctioneer's payoff in problem $1'$ depends on the profile of mixed strategies $\rho = \rho(\theta_i)_{\theta_i \in \mathbf{I}}$ only through finitely many indicators, one for each type $\theta \in \mathbf{\Theta}$, of the form $\int_{I(\theta)} \int_{\mathbf{\Omega}} \left(\bar{b}(\theta_i, \omega, \bar{p}) - e^{\theta_i} \right) d\rho(\theta_i) dv$. Given a mixed strategies equilibrium profile ρ, there exists a profile $(\bar{\omega}(\theta_i))_{\theta_i \in I(\theta), \theta \in \mathbf{\Theta}}$ such that the Dirac measure $\hat{\rho}(\theta_i)$ at $\omega(\theta_i)$ is an extreme point of the set $R(\theta_i)$, which is the consumer θ_i's best response to the prices chosen by the auctioneer in the previous equilibrium in mixed strategies. Moreover, $\int_{I(\theta)} \int_{\mathbf{\Omega}} \bar{b}(\theta_i, \omega, \bar{p}) d\rho(\theta_i) dv$ is the same as $\int_{I(\theta)} \int_{\mathbf{\Omega}} \bar{b}(\theta_i, \omega, \bar{p}) d\hat{\rho}(\theta_i) dv$.[31] Hence, we can replace $(\bar{\omega}(\theta_i))_{\theta_i \in I(\theta), \theta \in \mathbf{\Theta}}$ by $(\hat{\rho}(\theta_i))_{\theta_i \in I(\theta), \theta \in \mathbf{\Theta}}$ and keep all the equilibrium conditions satisfied. The indicators that the atomic auctioneer takes as given evaluated at $\hat{\rho}$ are still the same as when they were evaluated at ρ. Therefore, $\hat{\rho}$ is a degenerate equilibrium profile.

Remark 2: If we had assumed that consumers of the same type have the same preferences for private goods, then we would have a common best response $b(\theta, \omega)$ instead of one $b(\theta_i, \omega)$ for each consumer θ_i of this type. Then, Schmeidler's [35] purification result could be applied, since we could write $\int_{I(\theta)} \int_{\mathbf{\Omega}} (b(\theta_i, \omega(\theta_i))) \, d\rho(\theta_i) dv$ as $\int_{\mathbf{\Omega}} (b(\theta, \omega)) \, d(\int \rho(\theta_i) dv)$. In that case, we should check that the Price auctioneer's objective function Ψ is continuous with respect to the convergence in distribution: if a sequence of pure strategies profiles ω_n converges to ω is such $\int_{\mathbf{I}} g(\omega(i)_n) dv$ converges to $\int_{\mathbf{I}} g(\omega(i)) dv$, for any bounded continuous transformation g of ω, then $\Psi(\omega_n)$ converges to $\Psi(\omega)$. $\blacksquare$

Proposition 2: *An equilibrium for the generalized game is a competitive equilibrium.*

Proof. Let us consider our generalized game for an economy with lump-sum

[31] Observe that we could have written $\int_{\mathbf{\Omega}} \bar{x}(\theta_i, \omega) d\rho(\theta_i)$ instead of $\sum_{\omega_j \in \mathbf{\Omega}} \bar{x}(\theta_i, \omega_j) \rho(\theta_i, \omega_j)$ since the two are equivalent.

transfers and such that the consumption allocations in $\mathbf{X}$ have an upper bound $\mathbf{Z}$ that exceeds the attainability upper bound by an arbitrary small amount.

Let $\left(\bar{x}(\theta_i), \bar{g}_\omega, \bar{n}^\theta_\omega, \bar{y}_\omega, \bar{t}^\theta_\omega, \bar{\alpha}^\theta_\omega, \bar{p}\right)_{\theta_i \in I(\theta), \theta\in\mathbf{\Theta}, \omega\in\mathbf{\Omega}}$ be an equilibrium of the generalized game. We claim that this equilibrium is a free disposal equilibrium: there is no excess demand on the vectors of private goods. To see this, let us first aggregate consumers' budget constraints:

$$\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(p\left(x(\theta_i) - e^{\theta_i}\right) + t^{\theta_i}_\omega - \alpha^{\theta_i}_\omega - \phi^{\theta_i}_\omega \Pi^y_\omega\right) dv \geq 0$$

By noticing that all consumers of the same type in a jurisdiction type ω have the same lump sum tax rate ($t^{\theta_i}_\omega = t^\theta_\omega$), wage rate ($\alpha^{\theta_i}_\omega = \alpha^\theta_\omega$) and share of ownership in the private production ($\phi^{\theta_i}_\omega = \phi^\theta_\omega$), we can write

$$p\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(x(\theta_i) - e^{\theta_i}\right) dv + \sum_{\omega\in\mathbf{\Omega}} \sum_{\theta\in\mathbf{\Theta}} v(M(\omega,\theta))(t^\theta_\omega - \alpha^\theta_\omega - \phi^\theta_\omega \Pi^y_\omega) \geq 0$$

Now notice that we have $v(M(\omega,\theta)) \equiv n^\theta_\omega \mu_\omega$ by (MC.ii), so we can write

$$p\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(x(\theta_i) - e^{\theta_i}\right) dv + \sum_{\omega\in\mathbf{\Omega}} \mu_\omega \sum_{\theta\in\mathbf{\Theta}} (n^\theta_\omega t^\theta_\omega - n^\theta_\omega \alpha^\theta_\omega - n^\theta_\omega \phi^\theta_\omega \Pi^y_\omega) \geq 0$$

Since $\sum_{\theta\in\mathbf{\Theta}} n^\theta_\omega t^\theta_\omega = pc(g_\omega)$ (from the Poll tax and Transfers auctioneers' minimization problems), $\sum_{\theta\in\mathbf{\Theta}} n^\theta_\omega \phi^\theta_\omega = 1$ and $\Pi^y_\omega = py_\omega\left(\left(n^\theta_\omega\right)_{\theta\in\mathbf{\Theta}}\right) - \sum_{\theta\in\mathbf{\Theta}} n^\theta_\omega \alpha^\theta_\omega$, we have that

$$p\left(\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(x(\theta_i) - e^{\theta_i}\right) dv + \sum_{\omega\in\mathbf{\Omega}} \mu_\omega (c\left(g_\omega\right) - y_\omega\left(\left(n^\theta_\omega\right)_{\theta\in\mathbf{\Theta}}\right))\right) \geq 0$$

Now we check that in an equilibrium of the generalized game we have

$$\bar{f} \equiv \bar{p}(\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(x(\theta_i) - e^{\theta_i}\right) dv + \sum_{\omega\in\mathbf{\Omega}} (\mu_\omega (c\left(\bar{g}_\omega\right) - \bar{y}_\omega)) \leq 0$$

Suppose there is excess of demand of commodity $l \in \mathbf{L}$, that is,

$$\bar{f}_l \equiv p_l(\sum_{\theta\in\mathbf{\Theta}} \int_{I(\theta)} \left(\bar{x}^{\theta_i}_{l\omega} - e^{\theta_i}_l\right) dv + \sum_{\omega\in\mathbf{\Omega}} (\mu_\omega (c_l\left(\bar{g}_\omega\right) - \bar{y}_{l,\omega})) > 0$$

Then, the price auctioneer sets $\bar{p}_l$ equal to 1, but then the whole function becomes positive, $\bar{p}\bar{f} > 0$, a contradiction with the aggregation of the budget constraints. Moreover, the inequalities must hold as equalities (market clearing), $\bar{p}\bar{f} = 0$.

Otherwise, $\bar{p}_l$ would be equal to zero, which would make $\bar{x}_{l,\theta_i}$ hit the bound $\mathbf{Z}$, for every individual, a contradiction with feasibility. So condition (CE.4) of equilibrium is satisfied.

Now, $\bar{x}(\theta_i, \omega)$ is optimal for consumer θ_i, given the price vector $((\bar{\alpha}_\omega^\theta)_{\theta \in \mathbf{\Theta}, \omega \in \mathbf{\Omega}}$, $\bar{p}, (\bar{t}_\omega^\theta)_{\theta \in \mathbf{\Theta}, \omega \in \mathbf{\Omega}}$. Suppose it was not, say $\tilde{x}(\theta_i, \omega)$ is budget feasible at $(\bar{p}, \bar{\alpha}_\omega^\theta, \bar{g}_\omega, \bar{y}_\omega,$ $(\bar{n}_\omega^\theta, \bar{t}_\omega^\theta)_{\theta \in \Theta})$, and $u^{\theta^i}\left(\tilde{x}_\omega^{\theta_i}, \omega\right) > u^{\theta_i}(\bar{x}(\theta_i, \omega), \omega)$. By strict quasiconcavity of the utility function, $u^{\theta_i}\left(\delta \tilde{x}(\theta_i, \omega) + (1-\delta)\, \bar{x}(\theta_i, \omega), \omega\right) > u^{\theta_i}(\bar{x}(\theta_i, \omega), \omega)$, $\delta \in [0,1]$. Actually, when δ is sufficiently small the convex combination lies in $\mathbf{X}$ and

$$\bar{p}(\delta \tilde{x}(\theta_i, \omega) + (1-\delta)\, \bar{x}(\theta_i, \omega) - e^\theta) + \bar{t}_\omega^\theta + \frac{pc\left(\bar{g}_\omega\right)}{\bar{n}_\omega} \leq \bar{\alpha}_\omega^\theta + \phi_\omega^\theta \bar{\Pi}_\omega^y$$

a contradiction. So condition (CE.1.1) is satisfied. Condition (CE.1.2) follows from condition (GG.1). Condition (CE.2) follows by EPO condition and the equilibrium condition (GG.2). ■

Theorem 1 follows immediately from Propositions 1 and 2. Q.E.D.

Proof Theorem 2:

Proof. Suppose not, that is, there exists a blocking coalition $\tilde{I}$ and a feasible allocation $(\tilde{x}(\theta_i), \tilde{\omega})_{\theta_i \in \tilde{I}}$ offered by the jurisdiction managers of type $\tilde{\omega} = \left\{\tilde{g}_{\tilde{\omega}}, \left(\tilde{n}_{\tilde{\omega}}^\theta, \tilde{t}_{\tilde{\omega}}^\theta\right)_{\theta \in \mathbf{\Theta}}, \tilde{y}_{\tilde{\omega}}\right\}$, with $u^{\theta_i}\left(\tilde{x}(\theta_i), \tilde{\omega}\right) \geq u^{\theta_i}\left(\bar{x}(\theta_i), \bar{\omega}\right)$, $\forall \theta_i \in \tilde{I}$ and $u_{\tilde{\omega}}^{\theta_i}\left(\tilde{x}(\theta_i), \tilde{\omega}\right) >$ $u_{\bar{\omega}}^{\theta_i}\left(\bar{x}(\theta_i), \bar{\omega}\right)$ for at least one $\theta_i \in \tilde{I}$. The weak inequality $\geq$ implies $\bar{p}_0(\tilde{x}(\theta_i) - e^\theta) + \tilde{t}_{\tilde{\omega}}^\theta - \tilde{\alpha}_{\tilde{\omega}}^\theta - \phi_{\tilde{\omega}}^\theta \tilde{\Pi}_{\tilde{\omega}}^y \geq 0$, while the strict inequality $>$ implies

$$\bar{p}_0(\tilde{x}(\theta_i) - e^\theta) + \tilde{t}_{\tilde{\omega}}^\theta - \tilde{\alpha}_{\tilde{\omega}}^\theta - \phi_{\tilde{\omega}}^\theta \tilde{\Pi}_{\tilde{\omega}}^y > 0$$

Then aggregating over all consumers in the economy, we have

$$p\left(\left(\textstyle\sum_{\theta \in \mathbf{\Theta}} \int_{I(\theta)} \left(x(\theta_i) - e^{\theta_i}\right) dv\right) + \textstyle\sum_{\omega \in \mathbf{\Omega}} \mu_\omega (c\left(g_\omega\right) - y_\omega \left(\left(n_\omega^\theta\right)_{\theta \in \mathbf{\Theta}}\right))\right) > 0$$

a contradiction with the aggregation of the consumers' budget constraints. Pareto efficiency immediately follows by letting $\tilde{I}$ equal the whole population of consumers $\mathbf{I}$, and so the First Welfare theorem holds. ■

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