

TAXING BOUNDEDLY RATIONAL ENTREPRENEURS

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ABSTRACT. The note presents an evolutionary model of a population of entrepreneurs choosing between two production plans and thus trading opportunities. There is a tax system imposing income tax that is enforced on the population. Also, a level of the legal complexity of a tax system is modeled through boundedly rational choices of entrepreneurs. The model suggests that if the enforcement of a tax system is strong then the increase in the legal complexity leads to increase in the total tax revenue thus creating incentive for a governing body and politicians to introduce complicated unclear legal regulations.

1. INTRODUCTION

The reason for introducing a tax system is to collect revenue necessary for a governing body to fund goods provided to the general public like security or health services. However, there are costs associated with any tax system. These costs can be divided roughly into efficiency costs and collection costs. The larger of these two are efficiency costs resulting from the distortion of an allocation of resources relative to the optimal position, cf. Harberger (1966).

Without any doubt one of the more important features of a tax system is its complexity. There are many different measures of the complexity of a tax system. One example of a measure of the complexity of a tax system is the tax base. On the other hand the system of legal rules may itself be more or less complex. The more complex the tax system the more possibilities and incentives to evade or avoid taxes. This creates a tax gap and is related to the whole host of issues like mentioned already legal tax avoidance, illegal tax evasion but also offshore tax avoidance and tax havens or unintentional non-compliance.

Clearly, there is a relation between legal simplicity of a tax system and its efficiency in the sense of both collecting efficiency, enforcement and economic efficiency given the government budget requirements. It is not a surprise that there are strong connections between the politics and the tax law. These close connections led after the Schumpeter's theory of the tax, cf. Schumpeter (1954), to the vast body of research dubbed fiscal sociology or fiscal politics. This research posed one question among others: what is the impact of political institutions on attempts to reform taxation?

This short note focuses on the following question. Given strong enforcement, given tax rate and a tax base is it possible to increase tax revenues by merely increasing the legal complexity of a tax law? It is usually very hard for politicians to justify an increase in a tax rate to the general public. It is way easier to justify

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a new regulation. If the increase of a level of the legal complexity of a tax law could increase the expected tax revenues that would create an incentive for political bodies to create complicated tax law.

This paper offers a simple model within a framework of the evolutionary game theory, cf. Weibull (1997), that justifies such possibility. Section 2 provides the basic model and its general properties. Section 3 includes results and discussion. Section 4 concludes.

2. MODEL

There is a large population of entrepreneurs choosing between producing two complementary goods. At each time t an entrepreneur can choose to produce two units of the first good or two units of the second good. The share of the population choosing to produce the first good is denoted by x . The share of the population producing the second good is then $1 - x$.

Entrepreneurs are consumers also. An entrepreneur in the period t consumes either what he has produced or a mixed bundle of goods if he has managed to trade one unit of his good for one unit of the other good. At each period t entrepreneurs are matched randomly into pairs and if it happens that the matched entrepreneurs produce different goods then they exchange one unit of the goods.

After a period a single entrepreneur has a chance to switch to producing a different good. This scenario is repeated and the share x evolves in time. However, if the trade occurs the profit of each of the entrepreneurs is taxed according to the rate $c \in (0, 1)$.

The above process is modeled as a symmetric normal form game with the following payoff matrix

$$(1) \quad A = \begin{bmatrix} u_1 & u_1 + (1 - c)(1 - u_1) \\ u_2 + (1 - c)(1 - u_2) & u_2 \end{bmatrix},$$

where u_1 is the utility derived from consuming two goods of the first good and u_2 is the utility derived from consuming two units of the second good. The utility that comes from consuming a mixed bundle of goods is normalized to 1. Also, since the goods are assumed to be complementary the following inequality holds

$$(2) \quad 0 < u_2 < u_1 < 1.$$

Evolution of the share x depends on the decision process of entrepreneurs. This process comprises two steps. The first one is a choice of a strategy for comparison and the second is the choice between the pair of strategies. If q_{ij} is the probability that a single entrepreneur changes his strategy from i to j then

$$(3) \quad q_{ij} = p_i(x)r_{ij}(x)c_{ij}(x),$$

where p_i is the probability that a producer of the i -th good is chosen, r_{ij} is the probability that this entrepreneur chooses the j -th strategy for comparison and finally c_{ij} is the probability that the j -th strategy is chosen. All of these functions are assumed continuous on x . This process is called the pairwise comparison learning and has been reinvented in Sandholm (2010).

In the model concerned herein the pairwise comparison dynamic is hugely simplified. The following conditions are imposed. Firstly, a neutral representation of shares in a populations is assumed giving $p_1(x) = x$ and $p_2(x) = 1 - x$. This condition follows from the assumption that an entrepreneur with possibility to

change his strategy is chosen at random¹. That is, if a share x of a population produces the first good then a randomly picked entrepreneur produces the first good with the probability x .

Secondly, it is assumed that entrepreneurs follow the simple search mechanism as far as the choice of the comparison strategy is concerned. That is, it is assumed that an entrepreneur producing the i -th good chooses the j -th good for comparison with the probability $1/(S-1)$, where S is the total number of strategies. However, in the model concerned herein there are only two strategies and so $r_{ij}(x) = 1$. It is fair to mention here that there are other possibilities too. For example in the imitation dynamic a player selects a strategy for a comparison looking at the randomly chosen player and so $r_{ij} = x_j$ where x_j is a share of the population using the j -th strategy. This, however, requires a player to know what is the production strategy of a randomly picked producer. It is simpler to assume that an entrepreneur knows only all production possibilities and cannot check production strategies of other entrepreneurs.

Also, there is a deeper issue related to the very choice of a strategy for the comparison. The imitation model is rooted deeply within the mathematical biology where it reflects the very fact of replication. If a particular gene vanishes from a gene pool of a given population or a particular species becomes extinct in an ecosystem there is no way it can be replicated and so it remains extinct. This situation cannot be reverted. Mathematically it means that all faces of a simplex are forward invariant under any such dynamics including the famous replicator dynamics. On the other hand, actors in social environments tends to be extremely creative and able to recreate a strategy not used in a current state of a population. Therefore, it seems like the models similar to the simple search should capture better the behavior of human actors. A dynamics that allows for recreation of an extinct strategy is called innovative, cf. Ramsza (2009), and we will stick to such dynamics herein.

Finally, it is necessary to model the very choice of an entrepreneur between two possibilities, two production plans. As is common it is assumed here that this choices are based on the average payoffs w_i yielded by the respective production plans, where

$$(4) \quad \begin{aligned} w_1 &= u_1x + (u_1(1-c)(1-u_1))(1-x), \\ w_2 &= (u_2 + (1-c)(1-u_2))x + u_2(1-x). \end{aligned}$$

An entrepreneur should choose the more profitable option and so in fact his choice is based on the difference between the expected payoffs

$$(5) \quad \Delta w = w_1 - w_2 = -(1-c)(2 - u_1 - u_2)x + (1 - u_2) - c(1 - u_1).$$

If an entrepreneur had been perfectly rational his choice function would have been the standard best reply correspondence and so he would have chosen the higher ranking production plan with the probability 1. However, it is assumed that entrepreneurs are only boundedly rational and choose the higher ranking option with the higher probability yet it is possible that a worse option is chosen with some probability. Therefore, the choice functions are

$$(6) \quad c_{12}(x) = F_\gamma(-\Delta w(x, c)) \quad \text{and} \quad c_{21}(x) = F_\gamma(\Delta w(x, c)),$$

¹In fact it is assumed that players change their strategies according to opportunities following their private Poisson clocks. This is a standard assumption of evolutionary game theory, cf. Weibull (1997)

where F_γ is a cumulative distribution function (CDF) of a random variable with a density function f_γ . This model can be thought of as a slightly more general version of the smooth best reply.

For technical reasons² it is assumed that f_γ is continuous and has full support, that is $\text{supp}(f_\gamma) = \mathbb{R}$, and that entrepreneurs treat losses and gains in a symmetric fashion, that is f_γ is a symmetric function. Also, it is assumed that the CDF F_γ comes from a family indexed by a parameter $\gamma > 0$ and as $\gamma \rightarrow 0$ the function F_γ converges point-wise monotonically to the $\text{sgn}/2 + 1/2$ function. Also, it is assumed that as $\gamma \rightarrow \infty$ the function F_γ converges monotonically point-wise to $1/2$ over a closed interval $[u_2 - 1, 1 - u_2]$. One example of such a family of CDFs is the family of CDFs of normal random variables with zero mean and γ being the standard deviation σ . Figure 1(a) shows such a family of CDFs.

The boundedly rational behavior of entrepreneurs may be associated with a variety of reasons. However, as the only exogenous factor in the presented model that can modify Δw is the tax rate c it is assumed that the boundedly rational behavior is attributable to the legal complexity of the tax system modeled by the parameter $\gamma > 0$. The more complicated the tax system as far as the legal complexity is concerned the less discerning the entrepreneurs leading to flatter CDFs F_γ . On the other hand if a level of the legal complexity of a tax system converges to 0 then the CDFs F_γ converges to the best reply function.

Although, this way of interpretation has appeared before, for example in Spiegel (2006), we elaborate on the meaning of the CDF F_γ or the density function f_γ since there are at least two slightly different scenarios that can be understood. In the first scenario we assume the homogeneous population of players. Each player discerns between different production plans based on Δw but depending on a level of the legal complexity γ makes mistakes and chooses worse production plan with the probability $1 - F_\gamma(\Delta w)$. Consequently, on average the share of a population that makes the right choice equals $F_\gamma(\Delta w)$. The second scenario assumes a heterogeneous population, where given a level of the legal complexity γ the density function f_γ describes a distribution of the accuracy of judgment within the population. If the majority of the population is concentrated in the neighborhood of 0 then most of the population makes rational choices and the CDF F_γ is close to the best reply mapping. On the other hand if the density function f_γ has fat tails then it means that there is the substantial share of the population making irrational choices and the CDF becomes more flat³. All in all, a parameter γ models the legal complexity of a tax system driving the irrational choices of entrepreneurs because they are assumed to be only boundedly rational. Alternatively, the parameter γ could be potentially thought of as the very level of irrationality of entrepreneurs, however, this interpretation is more cumbersome and we rather choose the first one.

²It is possible to do without these assumptions. However, it complicates the model not introducing anything interesting

³In fact, there is yet another interpretation that assumes the homogeneous population that makes choices based on the noisy information about Δw where noise is given by the density function f_γ . This interpretation is in fact very close to the original development of the perturbed best reply mapping, cf. McFadden (1981)

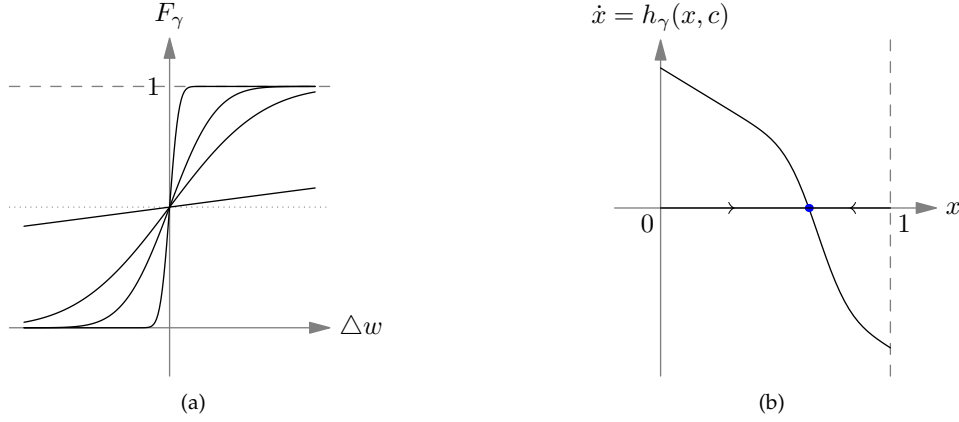


FIGURE 1. 1(a) An example of a family of CDFs F_γ – CDFs of normal random variables with zero mean and $\gamma = \sigma$ for various values of γ . As the legal complexity level γ decreases the functions converge to the best reply function. For high legal complexity levels entrepreneurs become indifferent between production plans. 1(b) Typical behavior of the dynamics (7). There is unique globally asymptotically stable equilibrium.

The deterministic differential equation approximating evolution of the share x is given by the following in-flow-out-flow equation (suppressing arguments)

$$(7) \quad \dot{x} = q_{21} - q_{12} = h_\gamma(x, c) = (1-x)F_\gamma(\Delta w) - xF_\gamma(-\Delta w).$$

The equation (7) is a specific example of a general theory on approximating Markov chains with deterministic differential equations, cf. Benaim & Weibull (2003). Any such dynamics is well defined, that is the simplex is forward invariant under any such dynamics. The existence of an equilibrium follows by the standard argument. Figure 1(b) shows typical behavior of $h_\gamma(x, c)$.

In what follows, Section 3, the main interest is the behavior of entrepreneurs and total tax revenue under the rising legal complexity level of a tax system. The basic behavior of the equations (7) is characterized by the following proposition.

Proposition 1. *Let (2) and all assumption on F_γ hold. Then for any $c \in (0, 1)$ there is the unique globally asymptotically stable equilibrium $\hat{x}(c, \gamma) \in (1/2, 1)$. \square*

Proof. Firstly, the difference Δw reads

$$(8) \quad \Delta w = -(1-c)(2-u_1-u_2)x + (1-u_2) - c(1-u_1)$$

and so it is a decreasing function of x for any $c \in (0, 1)$. Also, (7) can be rewritten as

$$(9) \quad \dot{x} = h_\gamma(x, c) = F(\Delta w) - x.$$

Simple differentiation leads to

$$(10) \quad \frac{\partial h_\gamma(x, c)}{\partial x} = -f_\gamma(\Delta w)(1-c)(2-u_1-u_2) - 1 < 0.$$

Consequently, there is a unique globally asymptotically stable equilibrium.

It is trivial to check that $\hat{x} < 1$. Suppose that $\Delta w = 0$ but then

$$(11) \quad x' = \frac{1}{1-c} \frac{(1-u_2) - c(1-u_1)}{2-u_1-u_2}$$

and consequently

$$(12) \quad h_\gamma(x', c) = -\frac{1+c}{1-c} \frac{u_1-u_2}{2-u_1-u_2} < 0$$

thus $\hat{x} < x'$ and $\Delta w > 0$ at the equilibrium. Rewrite (7) as

$$(13) \quad \frac{\hat{x}}{1-\hat{x}} = \frac{F_\gamma(\Delta w)}{F_\gamma(-\Delta w)}.$$

Since at the equilibrium $\Delta w > 0$ it follows from (13) that $\hat{x} > 1/2$. \square

The proof of Proposition 1 gives an upper bound for an equilibrium of the form

$$(14) \quad \frac{1}{2} < \hat{x} < \min \left\{ \frac{1}{1-c} \frac{(1-u_2) - c(1-u_1)}{2-u_1-u_2}, 1 \right\}$$

For c close to 0 this upper bound is quite precise.

3. DISCUSSION

This note focuses on three aspects of the presented model. Firstly, how increasing the tax rate c influences the market equilibrium \hat{x} . Secondly, how the legal complexity level γ of the tax system changes the market equilibrium \hat{x} . These two properties show how the expected total tax revenue curve $TR(c)$ shifts with changes of the legal complexity level γ .

Proposition 2. *Let $\gamma > 0$ be fixed and let $\hat{x}(c)$ be the market equilibrium. Then as the tax rate increases the equilibrium increases.* \square

Proof. Rewriting (7) as (9) leads at the equilibrium to the following equation

$$(15) \quad \hat{x}(c) = F_\gamma(\Delta w(\hat{x}, c)).$$

Differentiation of (15) with respect to c gives (suppressing arguments)

$$(16) \quad \begin{aligned} \frac{d\hat{x}}{dc} &= \frac{f_\gamma(\Delta w)((2-u_1-u_2)\hat{x} - (1-u_1))}{1 + f_\gamma(\Delta w)(1-c)(2-u_1-u_2)} \\ &> \frac{f_\gamma(\Delta w)((2-u_1-u_2)/2 - (1-u_1))}{1 + f_\gamma(\Delta w)(1-c)(2-u_1-u_2)} \\ &= \frac{f_\gamma(\Delta w)(u_1-u_2)}{2(1 + f_\gamma(\Delta w)(1-c)(2-u_1-u_2))} > 0 \end{aligned}$$

\square

Figure 2(a) shows behavior of the market equilibrium \hat{x} as a function of the tax rate c for various legal complexity levels γ of a tax system. As stated in Proposition 2 together with the increase of the tax level the market equilibrium shifts towards the production plan with the higher intrinsic value regardless of the legal complexity level of a tax system. This behavior is intuitive. The increase in the tax rate c makes trade less profitable and so more and more entrepreneurs switch to the production plan guaranteeing higher utility.

Although the share of entrepreneurs choosing the production plan with the higher intrinsic value is growing regardless of the legal complexity level of a

tax system the amount of the shift depends strongly on it. The higher the legal complexity level of a tax system the weaker the response. The next proposition addresses this question.

Proposition 3. *Let $\hat{x}(\gamma)$ be the market equilibrium and $\lambda = (1 - u_1)/(1 - u_2) \in (0, 1)$.*

- (1) *Then the market equilibrium decreases $\hat{x}(\gamma) \downarrow 1/2$ as the legal complexity level of a tax system increases $\gamma \uparrow \infty$ for any $c \in (0, 1)$.*
- (2) *Also, for any $\lambda \leq c < 1$ the market equilibrium converges $\hat{x}(\gamma) \uparrow 1$ as the legal complexity level decreases $\gamma \downarrow 0$. For any $0 < c < \lambda$ the market equilibrium converges*

$$\hat{x}(\gamma) \uparrow \frac{(1 - u_2) - c(1 - u_1)}{(1 - c)(2 - u_1 - u_2)}.$$

as the legal complexity level decreases $\gamma \downarrow 0$. □

Proof. Rewriting (7) as (9) leads to the following equation

$$(17) \quad \hat{x} = F_\gamma(\Delta w(\hat{x})).$$

As the right-hand side of (17) converges to $1/2$ the proof of the first claim is complete.

To prove the second point it is easier to rewrite (17) as

$$(18) \quad F_\gamma(\hat{y}) = \Delta w^{-1}(\hat{y}),$$

where

$$(19) \quad \hat{x} = \Delta w^{-1}(\hat{y}) = -\frac{\hat{y}}{(2 - c)(2 - u_1 - u_2)} + \frac{(1 - u_2) - c(1 - u_1)}{(1 - c)(2 - u_1 - u_2)}.$$

As the legal complexity level decreases $\gamma \downarrow 0$ then the function F_γ converges monotonically point-wise to the best reply function $\text{sgn}/2 + 1/2$. Therefore, the solution of (19) converges either $\hat{y} \downarrow 0$ for $c < \lambda$ or $\hat{y} \downarrow c(1 - u_2) - (1 - u_1)$ for $c \geq \lambda$. This in turn implies that

$$(20) \quad \hat{x} \uparrow \frac{(1 - u_2) - c(1 - u_1)}{(1 - c)(2 - u_1 - u_2)}$$

or that $\hat{x} \uparrow 1$ respectively. □

Figure 2(b) shows behavior of the market equilibrium \hat{x} as a function of the legal complexity level of a tax system γ for various values of the tax rate c . As stated in Proposition 3 for high levels of the legal complexity of a tax system the market shares of producers choosing either of the viable production plans are equal. This behavior is intuitive. High legal complexity level makes entrepreneurs less discerning as far as the differences in expected payoffs are concerned and in the end the choices become more and more random resulting in equal shares for both production plans.

It is quite different on the other extreme for low legal complexity levels of a tax system. As the legal complexity decreases the entrepreneurs start to realize the impact of the tax rate c and adjust to a proper market equilibrium. For high tax rates almost all entrepreneurs choose the production plan with the higher intrinsic value and so \hat{x} is close to 1. For lower tax rates there is a substantial share of entrepreneurs choosing either production plan.

Having studied the behavior of the market equilibrium as function of both the tax rate c and the legal complexity level γ it is time to look at the total tax revenue

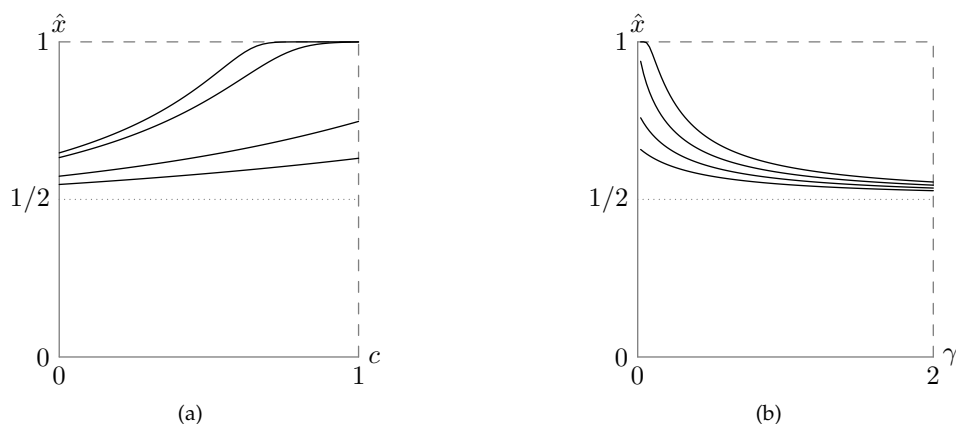


FIGURE 2. 2(a) Behavior of the market equilibrium against the tax rate c for various values of the legal complexity level γ . Rise in the legal complexity level γ shifts the curve downwards. 2(b) Behavior of the market equilibrium against the legal complexity level γ for various values of the tax rate c . Increase in the tax rate c shifts the curve upwards.

TR. In fact the actual number of trades at any given time is a random variable and so it is both simpler and common to talk about expected total tax revenue. Given the model the expected total tax revenue at the equilibrium \hat{x} is given as

$$(21) \quad \text{TR}_\gamma(c) = 2\hat{x}(1 - \hat{x}) \cdot c((1 - u_1) + (1 - u_2)),$$

where the first term is the probability of actual trade and the second term is the tax collected. Figure 3 shows typical behavior of the expected total tax revenue curves for various legal complexity levels of a tax system.

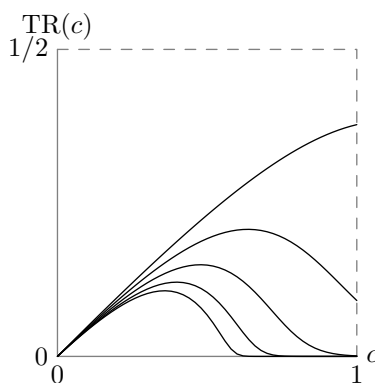


FIGURE 3. Typical behavior of expected total tax revenue for various legal complexity levels of a tax system. Higher legal complexity levels shift the curve upwards.

The TR curves for low legal complexity levels of a tax system are consistent with the idea of the Laffer curve, cf. Sachs & Larrain (1993). Let the legal complexity

level γ be fixed. For low tax rates the total revenue collected is low. Increases in the tax rate leads at the beginning to increases in the total tax revenue. However, after a certain point is passed the total tax revenue starts to fall. However, once the legal complexity level is high enough the total tax revenue does not have a maximum for $c < 1$. That is, as stated in the Proposition 3 the increase of the legal complexity level of a tax system shifts the total tax revenue curve upwards.

The reasons behind the described behavior are intuitive. If the tax system is simple then entrepreneurs can make precise judgments concerning the payoff differences between production plans. Any tax rate increase leads to lower profitability of trading thus forcing entrepreneurs to the production plan with the higher intrinsic value. And so there are less trades. However, for low tax rates the increase in the tax rate more than compensate for the lower number of trades. As the tax rate increases more and more entrepreneurs choose the first production plan and eventually almost all producer use the first production plan and the probability of a trade is practically zero thus making the total tax revenue fall to zero as well.

For the complex tax system behavior of entrepreneurs is practically random and so the response to increases in the tax rate is weak. Even for very high tax rates there is a substantial part of the population choosing suboptimal second production plan. This leads to ever rising total tax revenue curve.

There are at least two major components of tax transaction costs. These are efficiency costs and collection costs. The larger of these two are efficiency costs arising from the distortion of an allocation of resources relative to the optimal position, cf. Harberger (1966). In the model presented herein introduction of any taxes leads to the increase of the share of producers using the first production plan and so away from the no-tax market equilibrium. The collection costs can be further divided into compliance and administration costs.

Complexity of a tax system can be measured through a great variety of statistics but certainly including the tax base. The judgment what should be included in the tax base can be based on measures of consumption, income or wealth. In fact practically all tax system use a mixture of those three. However, in the model presented herein it is the legal complexity that is addressed⁴. The use of legal concepts, accounting terms and prescriptive rules reduce the ability of tax payers, entrepreneurs, to judge properly their options. In the model the whole complexity of the legal side of a tax system is hidden in the parameter γ . Increasing the legal complexity leads to suboptimal allocation of resource (and higher efficiency costs) but allows to collect higher revenues thus giving an incentive to a government. This is so especially in times of crisis that the total tax revenues increase through new regulations that may not increase the tax rate itself but rather increase the legal complexity of a system. This is even more so if the public opinion is taken into account. It is way easier to justify to the general public introduction of a new regulation, say a technical one, than an increase in the tax rate.

4. CONCLUSIONS

The model presented in this short note concerns the legal complexity of a tax system and how it influences the business choices of boundedly rational entrepreneurs.

⁴The tax base is constant as only the income is taxed from all trades.

The model itself is extremely simplified yet it captures the basic equilibrium relationship between the tax rate and the total tax revenue in a form of the simplified Laffer curve.

Once this relationship is established the influence of the legal complexity of the tax system on the position of the stylized Laffer curve is studied. The model predicts that any increase in the legal complexity of a tax system leads to higher efficiency costs due to a more suboptimal allocation of resources and higher tax revenues thus creating an incentive for a government.

The model itself is obviously simplified. It does not address a great variety of issues related to a tax system. In particular it does not address issues related to a tax gap like tax compliance, tax evasion, offshore avoidance and tax havens, administration and enforcement costs and many other. Also, the model is in fact static in that it addresses the behavior at the equilibrium rather than on a path leading to an equilibrium. All of the issues mentioned above could be incorporated into a more complicated model. In particular, it is possible to introduce a governing body controlling the legal complexity parameter γ together with a criterion functional depending on expected total tax revenues but also enforcement and administration costs changing the model into an optimal control problem. Continuing along this line it is also possible to introduce two additional pure strategies, one for each production plan, giving possibility of tax avoidance with payoffs depending on the government spendings on enforcement of a tax system. This and probably other extensions to the current model are left for future research.

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