Price Inference in Small Markets*

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Abstract

This paper studies information aggregation in markets for perfectly divisible goods with an arbitrary, possibly small, number of strategic traders. When trader valuations are heterogeneously correlated, a trade-off in information aggregation between small and large markets exists. As a result, small markets might offer better opportunities to learn from prices and be more efficient in aggregating information than large markets, in relevant economic environments. This paper provides the necessary and sufficient conditions for the monotonicity of price informativeness. The mechanisms underlying the trade-offs between small and large markets are characterized, and implications for market design are analyzed.

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1 Introduction

In many economic settings, traders do not perfectly know the value of the good traded, but each has some private information regarding that value. In these settings, markets are commonly viewed as serving a dual role. In addition to providing buyers and sellers with an

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opportunity to exhaust gains to trade, markets also allow traders to learn about the value of the good through a price mechanism, thereby improving their estimates and decisions. The literature on information aggregation has delivered strong results for large markets, where traders are approximately competitive. From rational-expectations or strategic models with a large number of traders,[1] we understand well the mechanisms through which large markets aggregate and transmit private information to market participants. In many markets, however, trade takes place among a small number of strategic traders. Important examples include financial markets such as intra-dealer markets, open-market operations by central banks, and numerous oligopolistic markets in which a few companies trade with a few supermarket chains. This paper shows that small markets may offer traders opportunities to learn from prices that large markets do not. Therefore, trade-offs in information aggregation and efficiency between small and large markets may exist.

To hint at the origin of the trade-off in information aggregation between small and large markets, consider the classical market setting: an arbitrary, possibly small, number of agents trade a perfectly divisible good, such as an asset, electricity, gold, emission permits etc. Each agent is uncertain about the true value of the good to him; a noisy signal provides an imperfect estimate of how much the good is worth to each agent. Suppose a new trader enters the market. In a now-larger market, would the remaining traders be able to better estimate their unknown values? With market design in mind, what is optimal for information aggregation market size? A new participant adds to the pool of information available in the market; the additional piece of information is useful in inference for other traders whenever values are correlated. The literature on small markets has focused on settings in which an underlying fundamental value of the good (modulo idiosyncratic shocks) determines the values of all traders. The existing small-market results show that markets are informationally efficient, in that all of the payoff-relevant information available in the economic system is revealed in prices (Dubey, Geanakoplos, and Shubik (1987); Vives (2009)). It follows that information brought by every additional trader is revealed in price, and that the informativeness of price to all market participants builds up as market grows. This paper contributes to the literature on information aggregation in markets with strategic traders by showing that these predictions may change dramatically in markets that feature more general interdependencies in values. In particular, when traders' values are correlated but heterogeneously so, traders may learn little from prices about their values, and smaller markets might provide superior learning opportunities.

A recent body of literature studies heterogeneity in information structures that stems

[1] Grossman (1989), Brunnermeier (2001) and Vives (2008) provide comprehensive reviews of this voluminous body of research.

from informational linkages: correlation of signal noisiness varies across pairs of traders (e.g., Colla and Mele (2010)). In these models, there is a fundamental value of a good and price is a sufficient statistic for the signals of all other traders. In this paper, heterogeneity derives from values rather than information—interdependence of values may differ across pairs of traders. As we show, price is typically not a sufficient statistic, which qualitatively changes the information aggregation properties. Heterogeneity in how trader values commove is inherent in many informationally decentralized markets or networks, and comovements in values within small groups of traders contain information that can potentially be useful in inference. Indeed, "local" shocks provide the sole source of learning from prices in some markets.[2]

We investigate interaction between the heterogeneity and learning from prices in a market modeled as a uniform-price double auction. All traders (buyers and sellers) are Bayesian and strategic. In a linear-normal setting, we analyze Bayesian Nash equilibria in which traders submit bid schedules that are linear in price. The model permits a rich class of interdependencies among agents' values of the traded good. This allows accommodating environments common in economic analysis, including: markets in which correlation among values varies with distance between agents; markets with value dependence within but not across groups of agents; or networks with size externalities on interdependence among values. Moreover, unlike models in which trader values derive from a common fundamental value, negative dependence of values is permitted here. We postpone the discussion of related literature until after a full development of our model.

We now summarize the main results. We begin by asking whether prices aggregate and convey to traders all private payoff-relevant information available in the market—if so, larger markets would always transmit more information to traders. We find that the information available in the market is aggregated if, and only if, correlation between values is the same for all pairs of traders; for instance, when it is induced by aggregate shocks that affect the fundamental value of the traded good. This holds in both small and large, limit markets. The result resonates with the conclusions of Vives (2009), who argued that strategic interactions (on one side of the market) do not obscure information aggregation. Our result complements the full-aggregation predictions of Vives by underscoring the role of heterogeneity in interdependence of trader values.

Given that markets do not generically aggregate all the available information, price informativeness about trader values need not improve with market size. We establish the

[2] E.g., Coval and Moskowitz (2001); and Harrison, Kubik, and Stein (2004). Veldkamp (2009) provides an overview of the empirical evidence.

necessary and sufficient conditions for the marginal contribution of a market participant to price informativeness to be positive. At the heart of the conditions lies interdependence of values: whether and how a new market participant impacts price informativeness depends upon the effect that his participation has on the comovement in values among all traders, measured by the average correlation. Price can become more informative with a new trader even if values become, on average, less aligned in the larger market. Further, while how much traders can learn through market is governed by the average correlation of values, the potential to learn outside of the market (from the information that is contained in signals but is not revealed in price) depends on heterogeneity in correlations. We find not only that prices in smaller markets may be more informative than in large markets, but also that small markets may be more efficient in aggregating total available information. We analyze and contrast how the optimal for price informativeness and informational efficiency market sizes vary with market primitives.

The informational inefficiency and the lack of monotonicity of price informativeness in many markets leads us to further inquire into the sources of the small markets' advantage in aggregating information. What component of information about values that is lost in large auctions is revealed in small auctions? We show that, in the considered class of auctions, individual values can be uniquely decomposed into a common value component (which need not result from an aggregate shock) and residuals. The qualitative difference in information aggregation properties between small and large markets is due to the ability of small—but not large—markets to partially aggregate "local" information contained in the residuals; large auctions reveal information contained in the common value component alone. Moreover, in certain environments, small auctions might offer opportunities to learn through noise.

Finally, we show that, for the class of preferences considered in this paper, the results on informativeness of equilibrium prices extend beyond double auctions to a larger class of models, competitive and strategic, including one-sided auctions.

The paper is organized as follows. Section 2 lays out the model of a double auction. Section 3 establishes the key properties of equilibrium. Section 4 provides an analysis of how information aggregation varies with market size, and evaluates the effectiveness of learning though markets. Section 5 investigates the determinants of trade-off in information aggregation between small and large markets. Section 6 extends the results to other models of market interactions. Section 7 concludes. Proofs of all results are contained in the Appendix.

2 A Model of Double Auction

Consider a market of a divisible good with $I \geq 2$ traders. We model the market as a double auction in the familiar linear-normal setting. Trader i has a quasilinear and quadratic utility function

$$U_i(q_i, m_i) = \theta_i q_i - \frac{\lambda}{2} q_i^2 + m_i, \tag{1}$$

where q_i is the obtained quantity of the good auctioned, m_i is money, and $\lambda > 0$. Each trader is uncertain about how much the good is worth to him. Trader uncertainty about the value of the good is captured by randomness of $\{\theta_i\}_{i \in I}$. The key novel feature of the model involves the richness of interdependencies among the intercepts of marginal utility functions $\{\theta_i\}_{i \in I}$, referred to as *values*, as described next.

Information Structure. Prior to trade, each trader i observes a noisy signal about his own true value θ_i, $s_i = \theta_i + \varepsilon_i$. Randomness in θ_i is interpreted as arising from shocks to preferences, endowment or any other shock that shifts the marginal utility of a trader. As long as the values $\{\theta_i\}_{i \in I}$ are correlated among agents, the signals of i's trading partners $\{s_j\}_{j \neq i}$ contain useful information about θ_i. The equilibrium price will be shown to reveal to each trader i some of such information about θ_i beyond the information contained in s_i. This gives rise to learning from market prices. To preserve the linearity of the model, we adopt the affine information structure: Random vector $\{\theta_i, \varepsilon_i\}_{i \in I}$ is jointly normally distributed, noise ε_i is mean-zero i.i.d. with variance σ_ε^2, and the expectation $E(\theta_i)$ and the variance σ_θ^2 of θ_i are the same for all i. The variance ratio $\sigma^2 \equiv \sigma_\varepsilon^2 / \sigma_\theta^2$ measures the relative importance of noise in the signal. The $I \times I$ variance-covariance matrix of the joint distribution of values $\{\theta_i\}_{i \in I}$, normalized by variance σ_θ^2, specifies the matrix of correlations,

$$\mathcal{C} \equiv \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,I} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1} & \rho_{I,2} & \cdots & 1 \end{pmatrix}. \tag{2}$$

Lack of any correlation among values corresponds to the *independent (private) value* (*IPV*) model, $\rho_{i,j} = 0$ for all $i \neq j$. At the other extreme, perfect correlation of values for all agents, $\rho_{i,j} = 1$ for all $i \neq j$, yields the *pure common value* model of a double auction (e.g., the classic model by Kyle (1989), which also has noise traders). In a recent contribution, Vives (2009) relaxed the strong dependence, while still requiring that the values of all trader pairs in the market commove in the same way, $\rho_{i,j} = \bar{\rho} \in [0, 1]$ for all $i \neq j$. This specification fits well markets in which there is an underlying fundamental value of the good that defines the values of all traders—a typical feature in many asset-pricing or macro models. Nevertheless, the specification precludes markets in which preferences of some agents commove more closely than others. Our model allows comovement of values $\rho_{i,j}$ to be heterogeneous across all pairs of agents in the market. To ensure that equilibrium price is equally informative across agents—which is necessary for the symmetry of equilibrium and, hence, tractability of the model—we require that, for each trader i, his value θ_i be *on average* correlated with other traders' values θ_j, $i \neq j$, in the same way,

$$\frac{1}{I-1}\sum_{j \neq i} \rho_{i,j} = \bar{\rho}, \tag{3}$$

for some $\bar{\rho} \in [-1, 1]$; that is, in each row (and column) in $\mathcal{C}$, the average of the off-diagonal elements is the same. $\bar{\rho}$ measures how a trader's value commoves on average with the values of all the other traders in the market. Given the restriction that the average correlation be the same across traders, $\bar{\rho}$ can be viewed as measuring the *commonality* in values of the traded good to market participants. We call the family of all auctions that satisfy condition (3) *equicommonal.* While restrictive, the class of equicommonal auctions subsumes a variety of economic environments beyond those with common $\rho_{i,j} = \bar{\rho}$. Let us introduce four examples of equicommonal auctions, which we use in the analysis to illustrate results. As a benchmark, we consider a model with identical pairwise correlations.

Example 1 *(*Uniform Correlations Model*) Values are equally correlated for all pairs of traders in a market; that is, $\rho_{i,j} = \bar{\rho}$ for all $i \neq j$.*

A stochastic process that results in uniform correlations and is often assumed in macro and finance literature involves aggregate shocks that determine (commovements in) preferences of all traders who are also subject to idiosyncratic shocks. The aggregate shocks could involve shocks to an aggregate endowment, monetary shocks, shocks in oil prices or other asset fundamentals.

In many markets, correlation in endowments or preferences varies with geographical or cultural proximity in a systematic way. This can be conceptualized in a spatial model in which correlation of values decays according to distance among agents.

Example 2 *(*Circle City Model*) I traders are located on a circle. The distance between any two immediate neighbors is normalized to one. Let $d_{i,j}$ be the shorter of the two distances between traders i and j (measured along the circle). To capture that values of closer neighbors commove more, correlation between any two traders $\rho_{i,j}$ is assumed to be decaying with distance, $\rho_{i,j} = \beta^{d_{i,j}}$, where $\beta \in (0, 1)$ is a decay rate.*

Often one can identify distinct groups of agents, such that correlation of values among members of a given group exceeds the average in the market. For instance, in markets with consumers and producers, (utility) parameters of consumers and (cost) parameters of producers tend to commove more strongly within than across the two groups.[3] Similarly, income, endowment or liquidity needs are all likely to be more correlated among traders from the same city (or country, or social network), rather than across different cities. This motivates the Twin Cities model.

Example 3 *(*TWIN CITIES MODEL*) There are two groups of traders of equal size, A and B, called cities (interpreted as neighborhoods, clubs, social affiliations etc.); the total number of traders adds up to an even number I. The values that members of each city derive from the good are perfectly correlated ($\rho_{i,j} = 1$); cross-group correlation can be positive or negative, or values can be independent ($\rho_{i,j} = \alpha;\ \alpha \in [-1, 1]$).*

Unlike the Circle City model, the Twin Cities model permits negative (average) correlation of values. A model with negative correlations $\alpha < 0$ accommodates interactions in which groups of traders compete for a pool of resources outside of the market (for instance, government transfers), and the division of the pool is uncertain during the trade stage. In the extreme case of $\alpha = -1$, the pool is fixed.[4]

Examples 1-3 all preserve symmetry in that, for each agent, the residual market is *ex ante* identical (assuming anonymity). The next example permits types of asymmetry that preserve equicommonality in the comovement of values.

Example 4 *(*ASYMMETRIC CORRELATIONS MODEL*) There are $I/3$ traders in city A, and $(2I)/3$ traders in city B, and I is a multiple of 3. Correlation of trader values differs between the two city: $\rho_{i,j} = 1$ if $i, j \in A$ and $\rho_{i,j} = (I-3)/(2I-3)$ if $i, j \in B$. For any pair of traders from distinct cities, values are uncorrelated ($\rho_{i,j} = 0$).*

The analysis is carried out at the level of correlations of values $\{\theta_i\}_{i \in I}$ specified by matrix $\mathcal{C}$, rather than underlying shocks that determine the joint distribution of values. Our results

[3] In Section 6, we extend the model with utility maximizers to double auctions with producers and consumers.

[4] Consider the following example. Let traders be characterized by a quasilinear utility function

$$U_i(q_i, m_i) = (q_i + t_i) - \frac{1}{2}\lambda (q_i + t_i)^2 + m_i, \tag{4}$$

where q_i is the quantity of a good obtained from trade in the market and t_i is the uncertain-at-the-time-of-trade transfer of a commodity determined by the government. This model gives rise to preferenecs as in (1), up to a constant, where $\theta_i \equiv 1 - \lambda t_i$. A model with a balanced government budget (a fixed pool of resources), $t_A = -t_B$, corresponds to $\alpha = -1$. In a model in which t_A and t_B are determined independently, $\alpha = 0$. Imperfect negative correlation of transfers gives rise to $\alpha \in (-1, 0)$.

hold for any data-generating process that gives rise to an equicommonal correlation matrix $\mathcal{C}$.

A Sequence of Auctions. Of central interest in the paper are market-size effects and the comparison between small and large markets. Therefore, instead of taking as the object of analysis an auction with a fixed number of traders, we analyze sequences of auctions indexed by the number of participants $\left\{A^I\right\}_{I=1}^{\infty}$. In all auctions in the sequence, the utility function remains unchanged. What changes is the number of traders I and, crucially, the equicommonal matrix $\mathcal{C}$ may vary with market size in an arbitrary way; the commonality $\bar{\rho}$ itself may change with market size, and so may other details of the correlation matrix. For a compact notation for a sequence of auctions, we define a measure of *market size* as a monotone function of the number of traders,

$$\gamma = \frac{I-2}{I-1}. \tag{5}$$

Index γ ranges between zero for $I = 2$ and one as $I \to \infty$. A sequence of auctions can now be conveniently summarized by the *commonality function* $\bar{\rho}(\gamma)$, which specifies commonality for any market size and can be a non-trivial function of γ. Throughout, we refer to auctions with $\gamma < 1$ as *small* and reserve the term *large* for limits as $\gamma \to 1$.

In the four examples above, the sequences of auctions are structured as follows. In the Uniform Correlations model, a new trader is neutral for commonality of values in that correlation of an entrant with each incumbent is equal to $\bar{\rho}$; the commonality function is given by $\bar{\rho}^{UC}(\gamma) = \bar{\rho}$. The Circle City model takes as a primitive the decay rate β and assumes that a new trader enlarges the market by increasing the circumference of the circle by one.[5] This allows analysis of market interactions where trader preferences become less and less alike as market expands. The commonality function is decreasing in market size, $\bar{\rho}^{CC}(\gamma) = 2(1-\gamma)\beta\left(1-\beta^{\frac{1}{2}\frac{1}{1-\gamma}}\right)/(1-\beta)$, assuming that I is an odd number. In the Twin Cities model, additional traders increase population in both neighborhoods, and the commonality function is given by $\bar{\rho}^{TC}(\gamma) = (\gamma + (2-\gamma)\alpha)/2$. Finally, in the Asymmetric Correlations model, new traders preserve the population ratio for the two cities and correlations; commonality $\bar{\rho}^{AC} = (2\gamma-1)/(6-3\gamma)$ is monotonically increasing to $\frac{1}{3}$.

[5] W.l.o.g. a trader can be added at an arbitrary position on a circle. Alternatively, one could assume that the circumference is fixed and that additional traders increase the density of the population. Such a formulation would imply that preferences commove more closely in pairs of traders in larger markets and, therefore, would not capture the decaying in distance commonality, which we intend to analyze.

Double Auction. We study double auctions based on the canonical uniform-price mechanism. Traders submit strictly downward-sloping (net) demand schedules, $\{q_i(p)\}_{i \in I}$; the part of a bid with negative quantities is interpreted as a supply schedule. The market-clearing price p^* is one for which the aggregate demand $Q(p) \equiv \sum_{i \in I} q_i(p)$ equals zero, $Q(p^*) = 0$. Trader i obtains the quantity determined by his submitted bid evaluated at the equilibrium price, $q_i^* = q_i(p^*)$, for which he pays $q_i^* \cdot p^*$. The trader payoff is given by the utility function (1) evaluated at (q_i^*, p^*). As a solution concept, we use the symmetric linear[6] Bayesian Nash equilibrium (henceforth, "equilibrium").

The Walrasian auction is commonly viewed as a natural model for the study of aggregation of information in markets—it allows traders to make trade choices that are contingent on prices (e.g., limit or stop orders); thus, even though the game is static and traders choose their strategies before (without) knowing the equilibrium price, traders can use information contained in prices by tailoring their choices to different prices and, hence, states of the world. Thus, a Walrasian auction allows studying the feedback between Bayesian updating and strategic considerations. This contrasts, for instance, with the Cournot competition in quantities, where traders can learn from prices but cannot incorporate the information conveyed by prices into their bids.

Finally, a noteworthy feature of our double-auction model is that all traders—buyers and sellers—are Bayesian and strategic in that they (endogenously) have price impact and take it into account in their trading decisions.[7]

3 Equilibrium in Double Auctions

This section characterizes equilibrium.

3.1 Sufficiency of Commonality

We begin by identifying a feature in the structure of equilibrium that will be useful in the analysis. Lemma 1 establishes that commonality $\bar{\rho}$ identifies equilibrium bids in that,

[6]The assumption that bids be strictly downward-sloping rules out trivial (no-trade) equilibria. In our model, "symmetric linear" means that bids have the functional form of

$$q_i(p) = \alpha_0 + \alpha_s s_i + \alpha_p p, \tag{6}$$

and that the coefficients α_0, α_s, and α_p are the same across traders.

[7]This distinguishes the present paper's model from those by Kyle (1989) and Vives (2009). As Kyle (1989) demonstrated, with pure common values, price is too informative for equilibrium to exist; to re-establish existence, Kyle lowered price informativeness by introducing noise traders, who are neither Bayesian nor take into account price impact. Vives (2009) considered a market with one-sided market power, with strategic Bayesian sellers and price-taking buyers.

ceteris paribus, the class of double auctions that share the same commonality $\bar{\rho}$ and have an otherwise arbitrary equicommonal correlation matrix have the same set of equilibria.

Lemma 1 *(*Sufficient Statistic for Equilibrium*) Let $\mathcal{C}$ and $\mathcal{C}'$ be two equicommonal correlation matrices with commonality $\bar{\rho}$. Then, other primitives being the same, a bid profile $\{q_i(\cdot)\}_{i\in I}$ is a symmetric equilibrium in a model with $\mathcal{C}$ if, and only if, it is a symmetric equilibrium in a model with $\mathcal{C}'$.*

For instance, if any two of the models from Examples 1-4 give rise to the same commonality $\bar{\rho}$, the equilibria are the same. That the commonality is a sufficient statistic for equilibrium derives from the fact that signals affect price only through their average,[8] which in turn comes from the symmetry of linear equilibrium bid functions (see equation (16) in Section 3.3). Lemma 1 allows one to derive equilibrium and fully characterize price informativeness for classes of equicommonal auctions with the same commonality, in abstraction from details of a correlation matrix other than statistic $\bar{\rho}$. We should stress that, while $\bar{\rho}$ identifies equilibrium, it does not determine all the properties of equilibrium. The same equilibria may exhibit different properties, depending on details of correlation matrices distinct from the commonality. In particular, heterogeneity in comovement of values across trader pairs will be shown as instrumental in determining informational (in)efficiency (Section 4.3).

3.2 Existence of Equilibrium

When traders learn from prices in a double auction, they shade their bids relative to the bids they would submit if the market were competitive and values were independent private ($\rho_{i,j}=0$ for $i\neq j$). The steepening of equilibrium bids arises from two considerations, the traders' price impact and the fact that price is informative about $\{\theta_i\}_{i\in I}$. As is well known (e.g., from Kyle (1989)), such steepening cannot be too strong for equilibrium to exist. Analogously, we derive an upper bound on commonality, which in terms of primitives is given by

$$\bar{\rho}^{+}\left(\gamma,\sigma^2\right)=\frac{\gamma^2-2(1-\gamma)\sigma^2+(1-\gamma)\sqrt{4\sigma^4+\left(\gamma\frac{2-\gamma}{1-\gamma}\right)^2}}{2\gamma}>0. \tag{8}$$

[8] Along with the linearity of the covariance operator, which implies that in an equicommonal auction, covariance of θ_i with the average signal depends on $\mathcal{C}$ only via the average correlation,

$$cov\left(\theta_i,\bar{s}\right)=\frac{1}{I}cov\left(\theta_i,s_i\right)+\frac{I-1}{I}\bar{\rho}\sigma_\theta^2. \tag{7}$$

For $\gamma = 0$, the upper bound is defined as the limit of (8) as $\gamma \to 0$, i.e., $\bar{\rho}^{+}(0, \sigma^2) \equiv 0$. Moreover, in no stochastic process can the average correlation $\bar{\rho}$ be smaller than the lower bound of[9]

$$\bar{\rho}^{-}(\gamma) = -\frac{1}{I-1} = -(1-\gamma) < 0. \tag{9}$$

Thus, only markets with a finite number of traders admit a negative commonality. Proposition 1 provides the necessary and sufficient conditions under which the linear Bayesian Nash equilibrium exists in equicommonal double auctions; in particular, commonality has to be *strictly* between the two bounds. The conditions are assumed thereafter in the analysis.

Proposition 1 (EXISTENCE OF EQUILIBRIUM)*In an equicommonal double auction, a symmetric linear Bayesian Nash equilibrium exists if, and only if,*

$$\bar{\rho}^{-}(\gamma) < \bar{\rho} < \bar{\rho}^{+}\left(\gamma, \sigma^2\right). \tag{10}$$

The symmetric equilibrium is unique.

Note that bounds (8) and (9) apply to the average correlation of values, and pairwise correlations in the auction can be arbitrarily high or low. For example, while equilibrium does not exist in double auctions with pure common values, the Twin Cities model with pure common values within groups of traders (and imperfectly correlated values across groups) admits equilibrium.

It is instructive to relate Proposition 1 to general existence results for the class of pure-strategy monotone Bayesian Nash equilibria. Specifically, explaining why Proposition 1 does not follow from these results highlights mechanisms distinctive to perfectly divisible-good double auctions. Take a simple example with two traders ($\gamma = 0$) and independent private values ($\bar{\rho} = 0$). By Proposition 1, equilibrium does not exist as the upper-bound condition is violated. The mechanism underlying the non-existence is the following: It can be shown that the slope of the best response to an arbitrary (inverse) bid of the opponent is strictly greater than the slope of the opponent's (inverse) bid. Hence, there does not exist a Nash equilibrium in which both inverse bids have finite slopes. However, vertical bids are not in the set of strategies, which comprises downward-sloping bid functions.[10] The mutual steepening is a robust feature of the uniform-price mechanism in games with demand

[9] $Var\left(\frac{1}{I}\sum_{i\in I}\theta_i\right) = \frac{1}{I^2}\left(I\sigma_\theta^2 + (I-1)I\bar{\rho}\sigma_\theta^2\right)$. $Var\left(\frac{1}{I}\sum_{i\in I}\theta_i\right) \geq 0$ if, and only if, $\bar{\rho} \geq \bar{\rho}^{-}(\gamma)$.

[10] More formally, the set of downward-sloping linear bid functions is not compact with respect to any topology for which payoffs are continuous. Compactness of strategy set is assumed by the available theorems for existence of non-trivial (trade) pure-strategy Bayesian Nash equilibria. While including vertical bids would close the strategy set, with vertical bids price is no longer defined by market clearing. Specifying an allocation rule to complete the definition of equilibrium would yield a no-trade equilibrium. This paper focuses on non-trivial equilibria.

functions as strategies. Additionally, when values are (on average) positively correlated, the mutual steepening of best responses is enhanced by price informativeness. The upper bound in condition (10) assures that both effects are moderate and that there exist bids with finite slopes that are mutual best responses. The lower bound assures that the conditional expectation exists.

By Lemma 1, other primitives being the same, all equicommonal auctions with the same statistics $(\gamma, \bar{\rho})$ have the same equilibria. The class of all such auctions can be represented as a point in the Cartesian product $[0,1] \times [-1,1]$.[11] Condition (10) specifies upper and lower bounds on commonality for any market size. In Figure 1A, for all auctions located on and above curve $\bar{\rho}^{+}(\gamma, \sigma^2)$, price is too informative for equilibrium to exist. There are no auctions that are represented by points located below bound $\bar{\rho}^{-}(\gamma)$; for points on $\bar{\rho}^{-}(\gamma)$, the conditional expectation does not exist. The two bounds are tight—for any pair $(\gamma, \bar{\rho})$ that satisfies the strict inequalities (10), one can find an auction for which equilibrium exists. A sequence of equicommonal auctions $\left\{A^I\right\}_{I=1}^{\infty}$ is represented by the corresponding commonality function $\bar{\rho}(\gamma)$, which keeps track of how the varying details of the correlation matrices in the sequence convert into changes in commonality (Figure 1B).

Proposition 1 has the following implications for the equicommonal models introduced in Section 2. With two traders, equilibrium fails to exist in the Uniform Correlations model with non-negative $\bar{\rho}$, and in the Circle City model. In the Twin Cities model with $\alpha = -1$, equilibrium does not exist, regardless of the number of traders. (Sometimes, we will consider a model with $\alpha \simeq -1$, by which we mean the limit as $\alpha \to -1$.)

3.3 Equilibrium Inference and Bids

When estimating his value θ_i, a trader uses statistical information contained in his private signal s_i and equilibrium price p^*. This section characterizes how the inference is reflected in equilibrium bids.

Given the linearity of equilibrium bids and the affine information structure, for each trader i, the posterior of θ_i conditional on p^* and s_i is normally distributed and fully described by the first two moments. The conditional expectation is linear in p^* and s_i. As shown in the Appendix, the equilibrium price is equal to $p^* = (1/I)\sum_{i \in I} E(\theta_i | s_i, p^*)$. By the law of iterated expectations, the unconditional expected price is equal to $E(p^*) = E(\theta_i)$,

[11] More precisely, γ takes values from a countable subset of $[0,1]$ given by

$$\Gamma \equiv \{\gamma \in [0,1] | \gamma = (I-2)/(I-1) \text{ for } I = 2, 3, ...\}. \tag{11}$$

Commonality function maps $\bar{\rho}(\cdot) : \Gamma \to [-1,1]$. The space of equicommonal auctions is given by a Cartesian product $\Gamma \times [-1,1]$.

and the posterior expectation $E(\theta_i|s_i, p^*)$ can be written as

$$E(\theta_i|s_i, p^*) = c_\theta E(\theta_i) + c_s s_i + c_p p^*, \tag{12}$$

where $c_\theta = 1 - (c_s + c_p)$.

Substituting for expectation from (12) into price p^*, and using that in the symmetric equilibrium inference coefficients c_p and c_s are common to all traders, gives the price

$$p^*(s) = \frac{1-(c_p+c_s)}{1-c_p} E(\theta_i) + \frac{c_s}{1-c_p}\bar{s}, \tag{13}$$

where $\bar{s} \equiv \frac{1}{I}\sum_{i\in I} s_i$. In a symmetric equilibrium, price $p^*(s)$ is a deterministic function of—and hence perfectly reveals—the average signal $\bar{s}$. The expectation and the variance of the normal vector (θ_i, s_i, p^*) can then be expressed in terms of inference coefficients c_p and c_s and the primitive parameters. The projection theorem then allows finding the parameters of the model (12) in closed form (see Appendix),

$$c_s = \frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}, \tag{14}$$

$$c_p = \frac{(2-\gamma)\bar{\rho}}{1-\gamma+\bar{\rho}} \frac{\sigma^2}{1-\bar{\rho}+\sigma^2}. \tag{15}$$

Given commonality $\bar{\rho}$, market size γ itself also shapes inference coefficients c_s and c_p, because the number of traders determines the number of signals, which, in turn, affects price informativeness. Having endogenized the traders' model of expectations, fully described in terms of market primitives by (12) with (14) and (15), one can derive equilibrium bids.

Proposition 2 *(*Equilibrium Bids*) The equilibrium bid of trader i is*

$$q_i(p) = \frac{\gamma - c_p}{1-c_p}\frac{c_\theta}{\lambda} E(\theta_i) + \frac{\gamma - c_p}{1-c_p}\frac{c_s}{\lambda} s_i + \frac{\gamma - c_p}{1-c_p}\frac{1}{\lambda} p. \tag{16}$$

4 Information Aggregation

In this section, we address the main question of the paper: How does the size of a market affect the market's ability to aggregate information dispersed among traders? A logically prior question is that of informational efficiency of markets; for if markets aggregated all the available information, larger markets would convey to traders more information than smaller markets. We show that, generically, markets are not efficient (Section 4.1). We then examine how much payoff-relevant information about θ_i is conveyed in price and give conditions under which price informativeness increases with a new market participant (Section 4.2). Finally, we analyze how much information is lost in the aggregation process and how the loss depends on the correlation matrix, as well as the size of the market (Section 4.3).

4.1 Informational Efficiency

As such, the overall information available in a market increases with every new trader. In informationally efficient markets, information contained in the traders' signals is fully incorporated in price, and price informativeness builds up as market grows. We now characterize which equicommonal markets are informationally efficient. The question about informational efficiency of markets is important in its own right and has attracted attention of researchers at least since the debate between Hayek and Lange. That markets aggregate private information dispersed among traders, which is not available to the planner, was one of Hayek's key arguments in favor of free (large) markets.

Following the tradition from the rational-expectations literature, we conceptualize informational efficiency by means of a privately[12] revealing price. Setting a market against a benchmark where each agent learns his value θ_i would be too stringent an efficiency criterion, as such information is not available in the economic environment. The total available information corresponds to the profile of all signals of all traders, $s \equiv \{s_i\}_{i \in I}$; it "might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process" (Hayek, 1945, p. 526). Every Bayesian player i, who also observes his own signal s_i, can learn about his value θ_i from a privately revealing price as much as he would if he had access to all the information available in the market, s.

Definition 1 *Equilibrium price is* privately revealing *if, for any trader i, the conditional distribution of the posterior of θ_i satisfies*

$$F(\theta_i | p^*, s_i) = F(\theta_i | s) \tag{17}$$

for every state s and the corresponding equilibrium price $p^* = p^*(s)$.

If price conveys all the available information to market participants, the signals of other agents contain no useful information for trader i beyond the information contained in price and his own signal. Proposition 3 characterizes which double-auction settings are efficient in this sense.

[12] "Privately" indicates that price reveals all the available information for trader i if combined with his signal s_i.

Proposition 3 *(*Aggregation of Private Information*) In a small double auction, the equilibrium price is privately revealing if, and only if, the model has uniform correlations, i.e.,* $\rho_{i,j} = \bar{\rho}$ *for all* $i \neq j$.

Proposition 3 extends to large auctions as long as $\lim_{\gamma \to 1} \bar{\rho}(\gamma) < 1$. Our result is consistent with that of Vives (2009), who examined (small and large) markets with uniform correlations, and proved the positive part of our result.[13] Even if learning through market does not suffice for traders to learn their values, with uniform correlations, they learn all that is available.

Let us emphasize that the lack of full private revelation of information in Proposition 3 does not result from a presence of noise traders (e.g., Kyle (1989)), or uncertainty about aggregate endowment, or uncertainty which has dimension greater than that of price (e.g. Jordan (1983)). In particular, in any equicommonal auction, for any agent, there exists a statistic that is sufficient for the payoff-relevant information contained in the signals of other agents. The sufficient statistic is given by a properly weighted average signal, where the weights depend on correlations $\mathcal{C}$ and may differ across traders. Thus, the dimension of the sufficient statistic, which is equal to one, exactly matches the dimension of the learning instrument (price). However, in an equicommonal auction, price deterministically reveals the (equally weighted) average signal $\bar{s}$, which is a sufficient statistic for all agents only in models with uniform correlations.

4.2 Price Informativeness

Consider a market with I traders and suppose that a new trader enters. Would each of the I traders be able to better estimate his value θ_i in the larger market? The signal of a new trader brings additional information to the system and, whenever values are correlated, the new information is payoff-relevant for other market participants and valuable in inference. We examine under what conditions the increased pool of available information translates into a more informative market price. As we demonstrate, the informativeness of equilibrium price need not be monotone in the number of traders in relevant economic settings. For instance, in the spatial Circle City model, a new trader improves inference from prices when the city is small but not when it is large. In the Twin Cities model with $\alpha \simeq -1$, price informativeness decreases with every additional trader and, hence, learning from prices can be most effective in the smallest market.

[13] Vives (2009) studied a model with one-sided market power. In Section 6, we show that results from this paper extend beyond double auctions to markets with one-sided market power.

(The Lack of) Monotonicity in Price Informativeness. We provide general conditions that allow assessment of the marginal impact of a new trader on the informativeness of equilibrium price. To measure price informativeness, we look at how much inference through market—that is, conditioning on the equilibrium price p^*, as well as one's own signal s_i—reduces the variance of the posterior of θ_i conditional only on the signal,

$$\psi^+ \equiv \frac{Var(\theta_i|s_i) - Var(\theta_i|s_i, p^*)}{Var(\theta_i|s_i)}. \tag{18}$$

For all values of primitives, $\psi^+ \in [0, 1]$. No reduction in variance ($\psi^+ = 0$) occurs when price contains no payoff-relevant information beyond a private signal, whereas full reduction ($\psi^+ = 1$) is accomplished when price, jointly with signal s_i, precisely reveals the value of θ_i to trader i. In short, index ψ^+ quantifies the contribution of a market to inference about a trader value θ_i. In equicommonal auctions, the value of index ψ^+ is not trader-dependent.

For any sequence of equicommonal auctions $\{A^I\}_{I=1}^{\infty}$, define $\Delta\bar{\rho}(\gamma)$ as the change in commonality that results from adding a trader to an auction of size γ. $\Delta\bar{\rho}(\gamma)$ captures the impact of a trader on the average comovement of values of all traders in the market. Proposition 4 pins down the necessary and sufficient conditions for equicommonal auctions that guarantee that a new trader increases the informational content of price. All primitives of the auction but the correlation matrix are assumed to remain the same when the market is enlarged.

Proposition 4 *(Informational Impact) There exist thresholds $\tau^+_{\gamma,\bar{\rho}} < 0$ and $\tau^-_{\gamma,\bar{\rho}} > 0$ such that the marginal contribution of an additional trader to price informativeness is strictly positive if, and only if,*

$$\begin{aligned} \Delta\bar{\rho}(\gamma) &> \tau^+_{\gamma,\bar{\rho}} \text{ for } \bar{\rho}(\gamma) > 0, \\ \Delta\bar{\rho}(\gamma) &< \tau^-_{\gamma,\bar{\rho}} \text{ for } \bar{\rho}(\gamma) < 0. \end{aligned} \tag{19}$$

The two thresholds are characterized in the Appendix. Proposition 4 has a compelling economic interpretation: Whether a new trader increases price informativeness depends precisely on the change in the comovement of values that his participation induces in the market. The marginal impact of a trader is positive, provided that his participation does not reduce the (absolute) average comovement of values too much. Further, the informational content of price is affected by how the details of correlations $\mathcal{C}$ change as the market grows only to the extent that a new trader alters the average correlation of values.

Proposition 4 can be intuitively explained by constructing a map of price-informativeness curves inscribed in Figure 1 (see Figure 2). For each value $\psi^+ \in [0, 1]$, let a ψ^+-curve consist of all profiles $(\gamma, \bar{\rho})$ that give rise to price informativeness equal to ψ^+. When price is perfectly uninformative ($\psi^+ = 0$) for any market size γ, as it is, for instance, in auctions with independent private values ($\bar{\rho} = 0$), the 0-curve coincides with the horizontal axis. When price is informative, for any $\psi^+ \in (0, 1)$, a ψ^+-curve consists of a positive and a negative segment, located in the positive and negative quadrants of $\bar{\rho}$, respectively. This reflects that traders can learn from prices in environments with positive and negative dependence among values. For the slope of ψ^+-curves, one can show that, *ceteris paribus*, higher price informativeness ψ^+ can be achieved by increasing the size of a market γ (and the pool of signals) with a given commonality, or by increasing the absolute value of commonality $\bar{\rho}$ for a given number of traders. Therefore, the positive (negative) components of ψ^+-curve slope down (up), ψ^+-curves located further away from the horizontal axis correspond to greater price informativeness, and the curve for the maximal price informativeness $\psi^+ = 1$ comprises one point $(1, 1)$.

Take an arbitrary sequence of auctions represented by a commonality function $\bar{\rho}(\gamma)$. For any auction $(\gamma, \bar{\rho})$ in the sequence, the conditions from Proposition 4 can be seen as the commonality function crossing the ψ^+-curve at point $(\gamma, \bar{\rho})$ from below if $\bar{\rho} > 0$ or from above if $\bar{\rho} < 0$. The thresholds $\tau^+_{\gamma,\bar{\rho}}$ and $\tau^-_{\gamma,\bar{\rho}}$ measure the change in commonality that is just sufficient to maintain the price informativeness constant.

A general lesson from Proposition 4 is that the informativeness of equilibrium price in double auctions may exhibit essentially arbitrary non-monotone behavior. Moreover, while arbitrary, the effectiveness of learning through market can be uniquely pinned down by tracing how changes in details of correlations between trader values translate into changes of commonality.

As a more specific implication of Proposition 4, suppose one wishes to determine the market size that maximizes price informativeness in an equicommonal auction. In any model in which the absolute value of commonality $\bar{\rho}(\gamma)$ is non-decreasing in market size, the larger the market, the more traders can learn about their values. This is, for instance, the case in the Uniform Correlations model. As Proposition 4 indicates, learning can improve with market size even if values become less aligned, when the reduced comovement in values is sufficiently compensated by new payoff-relevant information brought by an additional trader. In the Circle City model, where correlation weakens with distance, new traders enhance learning when the city is small, but when the city size exceeds a certain threshold, price becomes less and less informative as the city grows further. The informativeness of price attains the maximum for an intermediate city size. In the model of Twin Cities with $\alpha \in (-1/3, 1)$, commonality is positive and increasing and so is price informativeness.

When trader values are moderately distinct between the two cities ($\alpha \in (-1, -1/3)$), price informativeness is U-shaped in market size γ, and learning is most effective for the extreme market sizes. The Twin Cities with $\alpha \simeq -1$ is an example of a model in which equilibrium price is most informative in the smallest market. Notably, in this model, $\bar{s}$ equals the average noise $\bar{\varepsilon}$ and equilibrium price is independent from each trader's value θ_i. Still, $c_p \neq 0$ and bidders learn from prices in all but large auctions—what is there to learn from? In Section 5, we investigate the mechanisms underlying inference from prices, which allow us to explain these behaviors of price informativeness.

Price Inference in Large Markets. Our next result asserts that inference in the limit large equicommonal auctions may feature three qualitatively different outcomes: traders may learn perfectly their value, learn it partially, or learn nothing at all.

Corollary 1 *(*Price Informativeness in Large Markets*) In the limit large auctions, for any i, the equilibrium price:*

(1) is perfectly uninformative about θ_i if, and only if, $\lim_{\gamma \to 1} \bar{\rho}(\gamma) = 0$;

(2) reveals some information about θ_i if, and only if, $\lim_{\gamma \to 1} \bar{\rho}(\gamma) \in (0, 1]$; *and*

(3) deterministically reveals θ_i if, and only if, $\lim_{\gamma \to 1} \bar{\rho}(\gamma) = 1$.

Whenever commonality converges to zero in large markets, agents learn nothing about their values through market prices. This occurs, for example, in the Twin City model with $\alpha \simeq -1$ and also in the Circle City model. Given that new traders always add to the pool of payoff-relevant information, why does the price become uninformative in the large Circle City? Note that, for any decay rate $\beta \in (0, 1)$, the value of any trader in the city is strongly correlated only with a group of close neighbors, and is essentially independent from the values of distant traders. Similarly, a strong correlation of values in a neighborhood becomes negligible in a large city. With respect to how much traders can learn from prices about their values (but not in regard to informational efficiency, see Section 4.1), a large city operates effectively like an environment with independent private values, despite there being (possibly significant) payoff-relevant information available in the economic environment. (See also Section 5.)

4.3 Informational Gap

Proposition 3 shows that, typically, markets do not aggregate all the payoff-relevant information. So far, we have examined how much traders can learn about their values from the information that prices convey, relative to what they learn from private signals. The analysis suggests that, often, prices contribute little over private signals, and that learning from prices need not advance with market size. To assess the extent of inefficiency of learning through markets, we ask: Given the information potentially available in the market (s), how much can one improve upon markets by exploiting other, non-market modes of learning, such as direct communication?

To quantify the discrepancy in informativeness between the market and the benchmark in which the information in signals s is revealed, we look at how much a trader observing all signals (s) would reduce the variance of the posterior conditional on the variables accessible to the trader through the market (s_i, p^*),

$$\psi_i^- \equiv \frac{Var(\theta_i|s_i, p^*) - Var(\theta_i|s)}{Var(\theta_i|s_i)}. \tag{20}$$

Statistic $\psi_i^- \in [0,1]$ measures the *informational gap* with respect to the efficiency benchmark. Indices ψ^+, defined in (18), and ψ_i^- measure the contribution of the market to learning, and the potential to learn outside the market, respectively; $\psi_i^- + \psi_i^+ \leq 1$. By Proposition 3, markets are informationally efficient ($\psi_i^- = 0$) if, and only if, the model has uniform correlations. When $\psi_i^- > 0$, some of the information available in the market is lost in price, and learning directly from signals of other traders would provide trader i with valuable information.

We now contrast the extent of informational inefficiency in the models from Section 2 (see Figure 3). In the Uniform Correlations model, there is no efficiency loss. In the Circle City and Twin Cities models, learning from market versus non-market opportunities to learn becomes less and less effective as the market grows. The limit ($\alpha \simeq -1$) Twin Cities model provides a dramatic example. Here, if agents knew all signals in the large market, they would be able to discover their values; yet, price loses all the information contained in s.

Notably, as our examples illustrate, in economic environments with heterogeneously correlated values, the informational efficiency may decrease with the number of traders—small markets might be more efficient in transmitting payoff-relevant information than large competitive markets, and informational efficiency and the potential to learn outside of the market may change in a non-monotone, and arbitrary, fashion as market size changes. In light of our previous results, a more general insight is that price informativeness and informational gap are governed by distinct mechanisms. While how much traders can learn from the market (ψ^+) depends on the average comovement of values $\bar{\rho}(\gamma)$ alone, how far off markets are from efficiency (ψ_i^-) is determined by the heterogeneity in comovement of values. Thus, markets that share the same commonality schedule $\bar{\rho}(\gamma)$ may nonetheless differ in how efficiently they aggregate the information available in the signals, s.[14] Furthermore, with equicommonal $\mathcal{C}$, unlike price informativeness (ψ^+), inefficiency ψ_i^- may differ among traders.

In the three aforementioned examples, the informational gap is not idiosyncratic, for traders face an identical residual market. Clearly, the residual market is often trader-specific. To fix ideas, consider a market in which a small number of producers whose cost parameters are strongly correlated trade with a large number of consumers whose preferences commove weakly. Such interdependence can be captured by the Asymmetric Correlations model in which citizens of city A are interpreted as producers (see Section 6). Although commonality and price informativeness are the same for all producers ($i \in A$) and consumers ($i \in B$), the informational gap is not. Specifically, producers have more incentive to learn outside the market to improve the precision of their estimates than consumers (see Figure 3C). It is easiest to see why in large markets. The within-group correlation determines the potential to learn outside of market: in a large market, members of city A (where $\rho_{i,j} = 1$), but not B, would be able to perfectly estimate their value θ_i having access to the signals of other citizens in their city. In this sense, the within-group correlation is more important for the informational gap than the size of the group in the Asymmetric Correlations model.

5 Informational Trade-offs

The absence of monotonicity of price informativeness in general suggests that small markets may potentially aggregate information that is lost in large markets, and that there is a trade-off in information aggregation between small and large markets. What can traders learn in small markets that they cannot learn in large markets? This section uncovers the trade-off in price informativeness by separating the components of the available information being transmitted through prices in small and large auctions. As a starting point, observe that the equilibrium price in a large auction reveals the average value. In a sequence of auctions $\left\{A^I\right\}_{I=1}^{\infty}$, let A^∞ be the limit auction with the vector of values $\{\theta_i\}_{i=1}^{\infty}$. Define

$$\bar{\theta}^\infty \equiv \lim_{I\to\infty} \frac{1}{I} \sum_{i\in I} \theta_i. \tag{21}$$

[14] For example, the efficiency gap is zero in auctions in which the correlation matrix $\mathcal{C}$ evolves with the number of bidders such that the commonality schedule coincides with $\bar{\rho}^{CC}(\gamma)$ from the Circle City model or $\bar{\rho}^{TC}(\gamma)$ from the Twin Cities model and, for any given market size γ, all pairwise correlations $\rho_{i,j}$, $i \neq j$, are the same across bidder pairs.

In the large auction, price reveals no other information contained in the signals other than $\bar{\theta}^{\infty}$; $\bar{\theta}^{\infty} \sim \mathcal{N}\left(E\left(\theta\right), \sigma_{\theta}^{2}\bar{\rho}^{\infty}\right)$, where $\bar{\rho}^{\infty} \equiv \lim_{\gamma \to 1} \bar{\rho}\left(\gamma\right)$. To provide economic interpretation for $\bar{\theta}^{\infty}$ and to understand better what it captures for an individual trader, we argue that there is a sense in which the average value in a large auction, $\bar{\theta}^{\infty}$, represents a common value component in a large double auction (Section 5.1). We then show that if one defines the notion of a common value component of an auction appropriately, the bits of information that come to be revealed in small and large auctions can be identified (Section 5.2).

5.1 Common Value Component of an Auction

Putting our model and analysis aside for a moment, suppose that one would like to define a notion of a common value component for a profile of traders' values $\{\theta_i\}_{i=1}^{\infty}$ in a large auction—a random variable that captures the comovement in values of all traders in a large market. The following definition seems appropriate.

Definition 2 *A* common value component *is a random variable X such that for each trader i, θ_i can be decomposed into X and $R_i \equiv \theta_i - X$, with (i) $X \perp R_i$, and (ii) $R_i \perp R_j$ for all $i \neq j$.*

The second condition assures that X is a maximal common value component in that the residuals cannot be decomposed in a non-trivial way. Definition 2 captures value comovement in auctions with pure common values or, more generally, in the Uniform Correlations model. Unfortunately, a thus-defined common value component fails to exist in any auction outside this class. The problem arises because condition (ii) does not admit any heterogeneity in correlations of values across pairs of traders.[15]

Lemma 2 *(*Identification 1*) A common value component from Definition 2 exists if, and only if, the correlation matrix $\mathcal{C}$ in the large auction A^{∞} has uniform correlations, i.e., $\rho_{i,j} = \bar{\rho}^{\infty}$ for all $i \neq j$. Moreover, $X = \bar{\theta}^{\infty}$, where the common value component is unique up to a constant.*

To have a notion of a common value component that accommodates heterogeneous interdependencies in values, we relax the second condition and require that, for each trader

[15] Consider the Uniform Correlations model and suppose that correlation of valuations increases for a pair of traders. The common value component remains the same, as it is defined for a large market and the increased correlation for one pair of agents is inconsequential. Nevertheless, since the residuals in the pair are not independent, the two traders can learn from each other's residuals in addition to what they can learn from the common value component.

i, the residual R_i be on average independent from the average aggregate residual,

$$R = \lim_{I\to\infty} \frac{1}{I} \sum_{i\in I} R_i. \tag{22}$$

Definition 3 *A* common value component *is a random variable X such that for each trader i, θ_i can be decomposed into X and $R_i \equiv \theta_i - X$, such that (i) $X \perp R_i$, and (ii) $R_i \perp R$.*

The weaker independence condition (ii) in Definition 3 preserves the maximality of the common value component. The punch of Definitions 2 and 3 is the same for markets with uniform correlations; with the latter definition, however, a common value component exists and is identified up to a constant in all equicommonal auctions.

Lemma 3 *(*Identification 2*) A common value component from Definition 3 exists if, and only if, the correlation matrix $\mathcal{C}$ in the large auction A^∞ is equicommonal, i.e., $\lim_{I\to\infty 1} \frac{1}{I}\sum_{j\in I,,j\neq i} \rho_{i,j} = \bar{\rho}^\infty$ for all $i \neq j$. Moreover, $X = \bar{\theta}^\infty$, where the common value component is unique up to a constant.*

The notion of a common value component is useful, not only in identifying the trade-off but also in understanding what kind of market shocks can be accommodated by various correlation matrices $\mathcal{C}$. By Lemma 2, it is as if, in the microfoundation underlying the joint distribution of values $\{\theta_i\}_{i=1}^\infty$ that induces $\mathcal{C}$, there were only two types of shocks: some shocks affect all agents in the same way (aggregate shocks), and any other shocks in the data-generating process for $\{\theta_i\}_{i=1}^\infty$ must be idiosyncratic. Again, while such a shock structure fits many markets relevant for asset pricing or macroeconomics, it precludes other forms of interdependencies in values through local shocks. In light of Proposition 4, in markets that are affected only by aggregate and idiosyncratic shocks, there is no trade-off in inference between small and large markets, and larger markets offer superior learning opportunities.

By Lemma 3, equicommonal auctions extend the class of markets beyond those captured by the Uniform Correlations model, as follows. By permitting non-zero correlation in residuals, equicommonal auctions do not restrict shocks underlying the residuals to being idiosyncratic, which admits rich forms of local comovement in values. Additionally, unlike Definition 2, Definition 3 defines a common value component regardless of whether a particular stochastic process that generates values $\{\theta_i\}_{i=1}^\infty$ involves a shock that affects all agents in the same way.[16] Hereafter, the common value component is understood as in Definition 3.

[16]Consider the Twin Cities model with $\alpha = 0$, $X_A \sim \mathcal{N}(0,1)$ and $X_B \sim \mathcal{N}(0,1)$. Although no shock affects all agents, and the values are independent for any pair of traders from the two groups, the common

The common value component X represents the comovement of all values in large auctions. From a trader's perspective, its informational content about θ_i might vary from almost full revelation (which obtains only with pure common values for almost all traders) to nothing (in which case the component is deterministic). Among the examples from Section 2, X is deterministic in the models of Circle City and Twin Cities with $\alpha \simeq -1$, as well as the IPV model. The common value component is non-degenerate in the Uniform Correlations model with $\bar{\rho} > 0$, the Twin Cities model with $\alpha > -1$, and the Asymmetric Correlations model.

Correlation in residuals represents the comovements among values present in groups of bidders but not the market as a whole. The contribution of the local comovements picked up by residuals to commonality $\bar{\rho}(\gamma)$ is measured by the average correlation in residuals $\bar{\rho}_R \equiv (1/(I-1)) \sum_{j \neq i} (cov(R_i, R_j)/\sigma_\theta^2)$. For a given market size γ, the residuals are positively correlated if, and only if, local comovements increase commonality of values over the comovement of X with p^*, $\bar{\rho}_R(\gamma) = \bar{\rho}(\gamma) - \bar{\rho}^\infty$. The residuals are positively correlated in the Circle City model, negatively correlated in the Twin Cities and Asymmetric Correlations models and uncorrelated in the Uniform Correlations model. While in all the considered examples, the absolute value of correlation in residuals monotonically decreases in market size and correlation does not change the sign as the market grows, these need not hold in general.

5.2 Small versus Large Auctions

To investigate the sources of learning in small and large auctions, we put more structure on how correlation matrices change in a sequence of auctions. We assume that an auction A^{I+1} results from adding a bidder to an auction A^I, and the correlations among bidders from the auction A^I are preserved in A^{I+1}. Precisely, a random vector $\{\theta_i\}_{i=1}^I$ in auction A^I is a truncation of an infinite vector $\{\theta_i\}_{i=1}^\infty$ in the large auction. The common value component from the large auction $X = \bar{\theta}^\infty$ can then be interpreted as the common value component for a subset of bidders, as well.

In light of Lemma 3, in any equicommonal auction, a signal of trader i can be written as a sum of three independent random components

$$s_i = \underbrace{X + R_i}_{\theta_i} + \varepsilon_i. \tag{23}$$

value component from Definition 3 (but not Definition 2) exists and is given by $X \sim \mathcal{N}(0, \frac{1}{2})$. The residuals $R_i \sim \mathcal{N}(0, \frac{1}{2})$ are perfectly negatively correlated for any pair of traders from different cities and $R_i \perp R$ in the large auction.

To investigate the nature of price inference in small and large equicommonal auctions, we characterize how the updating of expectation of θ_i via price by a bidder i can be explained in terms of comovements of p^* with each $y = X$, R_i, ε_i. Taking conditional expectation of (23), we have that

$$s_i = E\left(X|p^*, s_i\right) + E\left(R_i|p^*, s_i\right) + E\left(\varepsilon_i|p^*, s_i\right), \tag{24}$$

for any price p^* and signal s_i. Thus, a Bayesian bidder i who makes inference about his value θ_i based on s_i and p^* can be interpreted as decomposing the signal he received, s_i, into conditional expectations of each X, R_i, and ε_i. The conditional expectation of value θ_i then corresponds to $E\left(\theta_i|p^*, s_i\right) = E\left(X|p^*, s_i\right) + E\left(R_i|p^*, s_i\right)$. Given this interpretation of inference in the model, price is informative if its realizations affect the decomposition (24) of s_i.

The stochastic relation between the equilibrium price and the three components of s_i in the model can be described by the projection

$$p^* = \beta_X X + \beta_R R_i + \beta_\varepsilon \varepsilon_i + \eta, \tag{25}$$

where η is a random variable that is independent from X, R_i, and ε_i. Since X, R_i, and ε_i are mutually independent, the coefficients in regression (25), which measure the comovement of y with price, are given by $\beta_y = cov(y,p)/Var(y)$. In the Appendix (Lemma 6), we show that $\beta_y \geq 0$ for $y = X, R_i$ and ε_i, even if $\bar{\rho} < 0$.[17]

Heuristically, the equilibrium price may change the decomposition of s_i in (24) through two countervailing effects. With $\beta_X, \beta_R > 0$, a high realization of price indicates high realizations of X and R_i, respectively, and the agent revises the expectation $E\left(X + R_i|p^*, s_i\right) = E\left(\theta_i|p^*, s_i\right)$ upwards; since $\beta_\varepsilon > 0$, a high realization of price indicates a high realization of ε_i and, given the relationship (24) and that s_i is fixed, the agent revises the expectation of θ_i downwards. Thus, the overall effect on the expectation $E\left(\theta_i|p^*, s_i\right)$ depends on the relative strength of the two effects, in turn determined by β_X, β_R, and β_ε. More formally, price affects the expectation $E\left(y|s_i, p^*\right)$ if coefficient $c_{p,y}$ in

$$E\left(y|s_i, p^*\right) = E\left(y\right) + c_{p,y}\left(p - E\left(p^*\right)\right) + c_{s,y}\left(s_i - E\left(s_i\right)\right) \tag{26}$$

[17] In fact, price is strictly positively correlated with ε_i in small auctions. This is because price is a linear and increasing function of the average signal, $\bar{s} = \frac{1}{I}\left(X + R_i + \varepsilon_i\right) + \frac{1}{I}\sum_{j \neq i} s_j$, which is increasing in each of the three components. In addition, if the common value component is non-degenerate, it can be shown that $\beta_X > \beta_\varepsilon > 0$ using that X is positively correlated with s_j. The magnitude of β_R varies in the (residual) commonality $\bar{\rho}_R$ and $\beta_R \in [0, \beta_X]$, where zero is attained for $\bar{\rho}_R = -(1-\gamma)$ and β_X is attained for $\bar{\rho}_R = 1$.

is not zero. Lemma 4 characterizes the inference coefficient $c_{p,y}$ in terms of coefficients β_y.

Lemma 4 *In the linear conditional expectation $E(y|s_i, p^*)$, the price coefficient is*

$$c_{p,y} = c \sum_{y' \neq y} \left(\beta_y - \beta_{y'}\right) \sigma_y^2 \sigma_{y'}^2, \tag{27}$$

where c is a positive constant that is the same for all y and σ_y^2 is the variance of the component y.

By (24), $c_p = c_{p,X} + c_{p,R} = -c_{p,\varepsilon}$. As is evident from (25), when $\beta_X = \beta_R = \beta_\varepsilon$, equilibrium price is a noisy (linear) transformation of s_i, and hence it does not contain any information about y beyond s_i. The necessary condition for the price to be informative about θ_i ($c_p \neq 0$) is that either $\beta_X \neq \beta_\varepsilon$ or $\beta_R \neq \beta_\varepsilon$; that is, that the two countervailing effects described earlier do not perfectly offset each other. When $\beta_X > \beta_\varepsilon$, a bidder tends to attribute a higher price to a higher realization of X rather than to a higher ε_i. Therefore, the conditional expectation of X is increasing in price given s_i. By an analogous argument, so is the conditional expectation of R_i if $\beta_R > \beta_\varepsilon$. Having recast Bayesian updating in terms of β_X, β_R, and β_ε, we next identify the sources of learning in the models from Section 2 and, more generally, discuss how the results about price informativeness and its monotonicity from Section 4 can be explained in terms of the comovements captured by β_y.

Large Auctions: In large auctions, $\beta_R, \beta_\varepsilon \to 0$ as the covariances of R_i and ε_i with price vanish, and Bayesian updating derives solely from the comovement of price with X. It follows from (27) that price is informative ($c_p = c_{p,X} > 0$) in large auctions only if the common value component is non-degenerate ($\sigma_X^2 \neq 0$), as it is in the models of Uniform Correlations, Twin Cities with $\alpha > -1$ and Asymmetric Correlations. The nature of learning in small markets is distinct from that in large markets. We next consider a class of auctions with a deterministic X, where traders do not learn from prices in large markets and the distinction is transparent.

Small Auctions with a Deterministic X: Price can be informative in small auctions even if traders learn nothing about their values θ_i from prices in large auctions. When X is deterministic, and hence $R_i = \theta_i$, price inference can take one of two forms: (net) *learning through residuals* ($\beta_R > \beta_\varepsilon$), as in the Circle City model; or *learning through noise* ($\beta_R < \beta_\varepsilon$), as in the Twin City model with $\alpha \simeq -1$ (cf. (27)). With a positive commonality $\bar{\rho} = \bar{\rho}_R$, the comovement of price with R_i exceeds the comovement of price with noise ε_i. Learning through residuals occurs if, and only if, $\bar{\rho}_R > 0$; then, $c_{p,R} > 0$. Similarly, learning through noise occurs if, and only if, $\bar{\rho}_R < 0$; then, $c_{p,R} < 0$ (see Lemma

6 in the Appendix). Interestingly, the Twin Cities model with $\alpha \simeq -1$ is an example in which price is independent from residuals ($\beta_R = 0$), and hence from each bidder's value θ_i, and there is no countervailing effect in learning; traders learn exclusively through noise. In the IPV model, note that both residuals and noise are correlated with price in small markets ($\beta_R = \beta_\varepsilon > 0$); however, unlike in the models of Circle City and Twin Cities, the comovements perfectly balance, and price is uninformative.

Small Auctions with a Stochastic X: In auctions where X is not degenerate, learning about θ_i through residuals is reinforced and learning through noise is counterbalanced by learning through the common value component. In the Twin Cities model with $\alpha > -1$, $\beta_R = 0$ and depending on β_X, β_ε, σ_X^2 and σ_ε^2, traders learn either through noise or through the common value component. In the Uniform Correlations model (and only there), the residuals are mutually independent (Lemma 2); therefore, $\beta_R = \beta_\varepsilon$ and traders learn only through the common value component, as $\beta_X > \beta_\varepsilon$.

Monotonicity of Price Informativeness: Explaining price informativeness in terms of the comovements captured by β_X, β_R, and β_ε also sheds light on the monotonicity of price informativeness in different markets.[18] Whether and why ψ^+ is monotone in γ depends on which of the components of s_i dominates inference in (27). If traders learn only through residuals or noise, then $\beta_R, \beta_\varepsilon \to 0$ in large markets and price informativeness eventually decreases in market size. This occurs, for example, in the Circle City model (learning through residuals) or in the Twin Cities with $\alpha \simeq -1$ (learning through noise). If the comovement of X with price is the principal source of information and $\beta_\varepsilon = \beta_R$, then price is more and more revealing as the market grows (as, for example, in the Uniform Correlations model). The U-shaped price informativeness ψ^+ in the Twin Cities model with $\alpha > -1$ can be understood in terms of the two countervailing effects that derive from $\beta_X > \beta_\varepsilon > \beta_R = 0$. If the variance of X is sufficiently small ($\alpha \in (-1, -1/3)$), then the comovement of X with price is too small to outweigh the effect of learning though noise in small markets. As the market grows and β_ε decreases, the difference $|\beta_R - \beta_\varepsilon|$ monotonically decreases, as well, and ψ^+ diminishes. In addition, for any $\alpha \in (-1, -1/3)$ there exists γ for which the two effects perfectly offset and price informativeness attains its minimum equal to zero. For all market sizes beyond this threshold, traders infer through common value component, and because $\beta_X - \beta_\varepsilon$ is monotonically increasing, so is ψ^+. In the model of Twin Cities with $\alpha \in (-1/3, 1)$, learning through common value component dominates for all γ, and ψ^+ increases in γ for any market size.

Summary: To recap, the sole source of price inference in large auctions is the common value

[18] Price informativeness ψ^+ depends not only on c_p but also on variances. Here, we focus on how market size affects c_p—the driving force behind the behavior of ψ^+ as a function of γ in the considered examples.

component. Transmitting "local" information contained in residuals or noise represents the advantage of small auctions over large auctions. The loss of information in large auctions occurs whenever a correlation matrix $\mathcal{C}$ features heterogeneity. The loss is especially stark in economic environments where (1) the common value component is deterministic, and (2) the residuals are positively or negatively correlated, and thus correlation among residuals is the exclusive source of inference about θ_i. Market size affects the comovements of price with the common value component, residuals and noise in market size in a different fashion. Changes in commovements determine how price informativeness varies with additional bidders, and might give rise to the non-monotonicity of price informativeness studied in Section 4.2.

6 Other Market Structures

Double auctions are commonly considered a proper setting for thinking about price formation; they often accurately describe actual markets; buyers as well as sellers are fully rational, Bayesian and strategic. We now extend the results established in the previous sections to a larger class of market structures in equicommonal, uniform-price settings. To make ours directly comparable with standard models of oligopolistic industry, we augment the model by introducing producers. Each producer j is characterized by a quadratic cost function

$$C_i(y_i) = \theta_i \cdot y_i + \frac{\lambda}{2} y_i^2. \tag{28}$$

To preserve the symmetry of the model, we retain the assumption that λ be the same for all traders, utility maximizers and producers. While we continue to assume that correlation matrix $\mathcal{C}$ is equicommonal, we relax the assumption $E(\theta_i) = E(\theta_j)$ for all $i, j \in I$. In all of the models considered, traders submit bidding schedules that are aggregated to determine the price by market clearing. By convention, submitted bids are (net) demands $q_i(p)$ and, thus, the equilibrium supply of producer i is $y_i^* = -q_i(p^*)$ and his equilibrium profit is $p^* y_i^* - C_i(y_i^*)$.

The class of models for which our results extend straightforwardly includes the following:

- A double (Walrasian) auction with I traders, who are either strategic (price-taking) utility maximizers or strategic (price-taking) producers who submit net demand schedules. The solution concept is symmetric linear Bayesian Nash (competitive rational expectations) equilibrium.

- A model of a share auction in which an auctioneer sells Q units of a divisible good to I strategic (competitive) utility maximizers who submit demand schedules (e.g., Wilson (1979)) or an auctioneer buys Q units of a divisible good from I strategic (competitive) producers who submit supply schedules. The solution concept is symmetric linear Bayesian Nash (competitive rational expectations) equilibrium.

- A model of oligopolistic industry with I strategic (price-taking) producers, who submit supply schedules, and face a deterministic linear demand. The solution concept is linear Bayesian Nash (competitive rational expectations) equilibrium.

We call this class of models—and the double auction—*uniform-price models.* In Lemma 5 and Corollary 2, "equilibrium" refers to symmetric linear Bayesian Nash equilibrium or competitive rational expectations equilibrium, as appropriate for a given model.

Lemma 5 *(*Uniform-Price Model*) Let $\mathcal{C}$ and $\mathcal{C}'$ be two equicommonal correlation matrices with commonality $\bar{\rho}$ in a uniform-price model. Then, other primitives being the same, a profile $\{q_i(\cdot)\}_{i\in I}$ is an equilibrium in a model with $\mathcal{C}$ if, and only if, it is an equilibrium in a model with $\mathcal{C}'$.*

The argument behind Lemma 5 is analogous to that in Lemma 1. One implication of Lemma 5 is that the results on existence of equilibrium established in models with uniform correlations carry over to all other equicommonal models with the same commonality.[19] In general, in the equicommonal, uniform-price models equilibrium exists if, and only if, $\bar{\rho}$ is bounded by some $\tilde{\rho}^{+}(\gamma,\sigma^2)$ and $\tilde{\rho}^{-}(\gamma)$ where the lower bound $\tilde{\rho}^{-}(\gamma)$ is as in (9) while the upper bound is model-specific and is weakly larger than (8).

We extend the results from Section 4 to the class of uniform-price models.

Corollary 2 *(*Information Aggregation in Uniform-Price Models*) Consider a symmetric uniform-price model in which equilibrium exists. Propositions 3 and 4, Corollary 1, and Lemmas 2 and 3 hold.*

The intuition behind this result is the following. In the considered class of uniform-price models, the equilibrium bids are linear and symmetric. The equilibrium price then is a deterministic function of the average signal $\bar{s}$, regardless of what the model-specific bids are like. Therefore, in all of the considered models, equilibrium price contains the same information. Our results from Section 4 about price informativeness hold not only qualitatively but also quantitatively in all uniform-price models. Thus, while various uniform price models may induce different equilibrium strategies or allocations, they share the properties of price informativeness that one can establish without relying on the exact functional form (or there being a closed-form solution) for the bidding strategy.

[19] For example, Vives (2009) provided conditions under which a linear Bayesian Nash equilibrium exists in a model of oligopolistic industry with uniform correlations, $\bar{\rho} \geq 0$.

7 Conclusions

This paper investigated price inference in markets in which the values that traders derive from the exchanged good are heterogeneously correlated. Compared to markets in which trader values commove solely through aggregate shocks, the information aggregation properties, including monotonicity of price informativeness and informational efficiency, differ qualitatively when interdependencies among trader values are heterogeneous.

Some modeling assumptions, namely the quadratic functional form for the payoffs and the affine information structure, were adopted for reasons of tractability. Our analysis should provide a useful benchmark for more general environments with a unique equilibrium. The main limitation of our analysis, resulting from the assumption of equicommonality, is that price is equally informative for all trades. Allowing some agents to learn more from prices than others is a promising, but also challenging, direction for future research.

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Appendix

The proofs of Lemma 1 and Proposition 1 are provided after the proof of Proposition 2.

Proof 1 ***Proposition 2*** *(*Equilibrium Bids*) In the symmetric linear equilibrium, bids have the functional form $q_i(p) = \alpha_0 + \alpha_s s_i + \alpha_p p$, where constants α_0, α_s, and α_p are the same across all traders. Given bids of traders $j \neq i$, trader i faces a residual supply with a slope μ and a stochastic intercept which is a function of signals of other traders. The best response of trader i to the residual supply is given by the first-order (necessary and sufficient) condition: for any p,*

$$E(\theta_n|s_i, p) - \lambda q_i = p + \mu q_i. \tag{29}$$

By condition (29) *and market clearing, equilibrium price is equal to $p^* = \frac{1}{I}\sum_{i\in I} E(\theta_i|s_i, p^*)$. Given an affine information structure, the conditional expectation is linear, $E(\theta_i|s_i, p) = c_\theta E(\theta_i) + c_s s_i + c_p p$, where coefficients c_θ, c_s and c_p are identical across traders and, by $E(\theta_i) = E(s_i) = E(p^*)$, the coefficients also satisfy $c_\theta = 1 - c_s - c_p$. It follows that the equilibrium price is given by*

$$p^* = \frac{c_\theta E(\theta_i)}{1 - c_p} + \frac{c_s}{1 - c_p}\bar{s}, \tag{30}$$

where $\bar{s} = \frac{1}{I}\sum_{i\in I} s_i$. Using (30)*, random vector (θ_i, s_i, p^*) is jointly normally distributed,*

$$\begin{pmatrix} \theta_i \\ s_i \\ p^* \end{pmatrix} = \mathcal{N}\left[\begin{pmatrix} E(\theta_i) \\ E(\theta_i) \\ E(\theta_i) \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov(\theta_i, p^*) \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & cov(s_i, p^*) \\ cov(p^*, \theta_i) & cov(p^*, s_i) & Var(p^*) \end{pmatrix}\right]. \tag{31}$$

The covariances in (31) *are given by*

$$cov(\theta_i, p^*) = \frac{1}{I}\frac{c_s}{1 - c_p}\left(1 + (I - 1)\bar{\rho}\right)\sigma_\theta^2, \tag{32}$$

$$cov(s_i, p^*) = \frac{1}{I}\frac{c_s}{1 - c_p}\left((1 + (I - 1)\bar{\rho}) + \sigma^2\right)\sigma_\theta^2, \tag{33}$$

and

$$Var(p^*) = \frac{1}{I}\left(\frac{c_s}{1 - c_p}\right)^2\left((1 + (I - 1)\bar{\rho}) + \sigma^2\right)\sigma_\theta^2. \tag{34}$$

Applying the projection theorem and the method of undetermined coefficients, one can find the inference coefficients c_s and c_p,

$$
\begin{aligned}
c_s &= \frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}, && (35)\\
c_p &= \frac{(2-\gamma)\,\bar{\rho}}{1-\gamma+\bar{\rho}}\frac{\sigma^2}{1-\bar{\rho}+\sigma^2}. && (36)
\end{aligned}
$$

Using (29)*, the equilibrium bid is*

$$q_i\,(p) = \frac{1}{(\lambda+\mu)}\left[c_\theta E\,(\theta_i) + c_s s_i + (1-c_p)\,p\right]. \tag{37}$$

In equilibrium, the residual supply of trader i (i.e., a horizontal sum of bids of traders other than i) has the slope $\mu = (1-\gamma)\,(\partial q_i\,(p)\,/\partial p)^{-1}$. Consequently, $\mu = \lambda\,(1-\gamma)\,/\,(\gamma - c_p)$. In terms of exogenous parameters, the equilibrium bids are

$$q_i\,(p) = \frac{(\gamma-c_p)\,c_\theta}{(1-c_p)\,\lambda}E\,(\theta_i) + \frac{\gamma-c_p}{1-c_p}\frac{1}{\lambda}c_s s_i + \frac{\gamma-c_p}{1-c_p}\frac{1}{\lambda}p, \tag{38}$$

where c_s, c_p are given by (35) *and* (36)*, and $c_\theta = 1 - c_s - c_p$.*

Proof 2 ***Lemma 1*** *(*Sufficient Statistic for Equilibrium*) As is shown in the proof of Proposition 2, equilibrium bids* (38) *depend on correlation matrix $\mathcal{C}$ only through $\bar{\rho}$ and, hence, equilibrium bids coincide in all auctions characterized by the same commonality $\bar{\rho}$.*

Proof 3 ***Proposition 1*** *(*Existence of Equilibrium*) The proof proceeds by verifying that the profile of bids* (38)*, $i \in I$, constructed in the proof of Proposition 2 constitutes an equilibrium with downward-sloping bids. The model admits downward-sloping demands in equilibrium if, and only if, $\mu > 0$ (or, equivalently, $c_p < \infty$) and $\mu < \infty$ ($\gamma > c_p$). By the former condition,*

$$\bar{\rho} \neq -\,(1-\gamma)\,. \tag{39}$$

Combined with the fact that $\bar{\rho} \geq -\,(1-\gamma)$ for an arbitrary random vector $\{\theta_i\}_{i\in I}$, condition (39) *implies the desired lower bound on commonality $\bar{\rho} > -\,(1-\gamma)$. The upper bound is derived from the condition $\gamma > c_p$, which by* (36) *is equivalent to*

$$\bar{\rho} < \frac{\gamma^2 - 2(1-\gamma)\sigma^2 + (1-\gamma)\sqrt{4\sigma^4 + \left(\gamma\frac{2-\gamma}{1-\gamma}\right)^2}}{2\gamma}. \tag{40}$$

For any commonality $\bar{\rho}$ that satisfies the two bounds, it is straightforward to verify that bids from Proposition 2 constitute a linear Bayesian Nash equilibrium.

Proof 4 ***Proposition 3*** *(*AGGREGATION OF PRIVATE INFORMATION*) (Only if) Assume that price is privately revealing, that is $F(\theta_i|s_i,p^*) = F(\theta_i|s)$ for every s and $p^* = p^*(s)$, or equivalently the first two moments of the conditional distributions coincide. Using that price is a deterministic function of the average signal, we have that $F(\theta_i|s_i,p^*) = F(\theta_i|s_i,\bar{s})$. By the projection theorem,*

$$E(\theta_i|s_i,\bar{s}) = c_0 + c \cdot s, \tag{41}$$

where $c = (c_{s_1}, c_{s_2}, ..., c_{s_I})$ is a vector of constants in which all entries $j \neq i$ are identical. That the equality $E(\theta_i|s_{i,}\bar{s}) = E(\theta_i|s)$ holds for all s implies that the coefficients multiplying each s_i are the same in both conditional expectations. We now show that identical for all $j \neq i$ coefficients c_{s_j} in $E(\theta_i|s)$ imply that matrix $\mathcal{C}$ has uniform correlations. Let $\mathcal{V} = \sigma_\theta^2\mathcal{C} + \mathcal{I}\sigma_\varepsilon$ be the variance-covariance matrix of signals $\{s_i\}_{i\in I}$, and $x = \{cov(\theta_i,s_j)\}_i$ be the column vector of covariances. By the positive semidefiniteness of $\mathcal{C}$, $\mathcal{V}$ is positive definite and, hence, invertible. Applying the projection theorem, coefficients $c \in R^I$ in expectation $E(\theta_i|s)$ can be found from the condition $c = x^T\mathcal{V}^{-1}$, which gives

$$x = \mathcal{V}c. \tag{42}$$

For any $j \neq i$, the j^{th} row of (42) is

$$cov(\theta_i,\theta_j) = c_{s_j}\sum_{j\in I} cov(\theta_i,\theta_j) + \left(c_{s_i} - c_{s_j}\right) cov(\theta_i,\theta_j) + \sigma_\varepsilon cov(\theta_i,\theta_i), \tag{43}$$

where we used that coefficients c_{s_j} are the same for all $j \neq i$. (43) gives

$$cov(\theta_i,\theta_j) = \frac{I-1}{1-\left(c_{s_j}-c_{s_i}\right)}c_{s_j}\bar{\rho} + \frac{c_{s_j}}{1-\left(c_{s_j}-c_{s_i}\right)}\sigma_\theta^2. \tag{44}$$

$cov(\theta_i,\theta_j)$ is the same across all pairs of agents; that is, the correlation matrix $\mathcal{C}$ has uniform correlations.

(If) Assume that the correlation matrix $\mathcal{C}$ has uniform correlations. We derive the first two moments of $F(\theta_i|s)$. Given the uniform correlations matrix $\mathcal{C}$, the variance-covariance matrix $\mathcal{V}$ can be written as

$$\mathcal{V} = \begin{pmatrix} a & b & ... & b \\ b & a & ... & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & ... & a \end{pmatrix}, \tag{45}$$

where $a = \sigma_\theta^2 + \sigma_\varepsilon^2$ and $b = \bar{\rho}\sigma_\theta^2$. Its inverse is given by

$$\mathcal{V}^{-1} = \begin{pmatrix} \tilde{a} & -\tilde{b} & ... & -\tilde{b} \\ -\tilde{b} & \tilde{a} & ... & -\tilde{b} \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{b} & -\tilde{b} & ... & \tilde{a} \end{pmatrix},$$

where

$$\tilde{a} = \frac{\sigma_\theta^2 + \sigma_\varepsilon^2 + (I-2)\,\rho\sigma_\theta^2}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^2 + (I-2)\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\rho\sigma_\theta^2 - (I-1)\,\rho^2\sigma_\theta^4}, \tag{46}$$

and

$$\tilde{b} = \frac{\rho\sigma_\theta^2}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^2 + (I-2)\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\rho\sigma_\theta^2 - (I-1)\,\rho^2\sigma_\theta^4}. \tag{47}$$

Assuming w.l.o.g. that $i = 1$, we can write $x^T = \sigma_\theta^2\,(1, \rho, \rho, ..., \rho)$. From the projection theorem, the coefficients in expectation $E\,(\theta_i|s)$ are equal to $c = x^T\mathcal{V}^{-1}$, which gives

$$\begin{aligned} c_{s_i} &= \frac{\sigma_\theta^4 + \sigma_\varepsilon^2\sigma_\theta^2 + (I-2)\,\rho\sigma_\theta^4 - (I-1)\rho^2\sigma_\theta^4}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^2 + (I-2)\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\rho\sigma_\theta^2 - (I-1)\,\rho^2\sigma_\theta^4}, &(48)\\ c_{s_j} &= \frac{\rho\sigma_\varepsilon^2\sigma_\theta^2}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^2 + (I-2)\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\rho\sigma_\theta^2 - (I-1)\,\rho^2\sigma_\theta^4}. &(49) \end{aligned}$$

We now argue that inference through $E(\theta_i|s_i, p^*(s))$, with coefficients derived in (35) *and* (36) *results in the same weighting of all individual signals as coefficients* (48) *and* (49)*. To see this, write the equilibrium price as a function of signals. The expectation becomes*

$$E(\theta_i|s_i, p^*(s)) = E(\theta_i) + \left[c_s + \frac{c_p c_s}{1-c_p}\frac{1}{I}\right][s_i - E(s_i)] + \frac{c_p c_s}{1-c_p}\frac{1}{I}\sum_{j\in I}\left(s_j - E\left(s_j\right)\right), \tag{50}$$

and hence $E(\theta_i|s_i, p^*(s)) = E(\theta_i|s)$ if, and only if,

$$\begin{aligned} c_{s_j} &= \frac{c_p c_s}{1-c_p}\frac{1}{I}, &(51)\\ c_{s_i} &= c_s + \frac{c_p c_s}{1-c_p}\frac{1}{I}. &(52) \end{aligned}$$

That conditions (51) *and* (52) *hold can be verified using* (35), (36), (48), *and* (49)*. This proves the equality of expectations $E(\theta_i|s_i, p^*(s))$ and $E(\theta_i|s)$. Next, we demonstrate the equality of variances in the conditional distributions $F(\theta_i|s_i, p^*)$ and $F(\theta_i|s)$. Let ψ^s be* *defined by* $Var(\theta_i|s) = (1-\psi^s)\,\sigma_\theta^2$*. By the projection theorem,*

$$\begin{aligned}
\psi^s &= c_{s_i} + (I-1)\rho c_{s_j} \\
&= \frac{\sigma_\theta^4 + \sigma_\varepsilon^2\sigma_\theta^2 + (I-2)\,\rho\sigma_\theta^4 - (I-1)\rho^2\sigma_\theta^4 + (I-1)\rho^2\sigma_\varepsilon^2\sigma_\theta^2}{\left[\sigma_\varepsilon^2 + (1+(I-1)\,\bar{\rho})\,\sigma_\theta^2\right]\left(\sigma_\theta^2\,(1-\bar{\rho}) + \sigma_\varepsilon^2\right)} \\
&= \frac{(1-\rho)}{(1-\bar{\rho}+\sigma^2)}\frac{1+\sigma^2+(I-2)\,\rho-(I-1)\rho^2+(I-1)\rho^2\sigma^2}{(1-\rho)\,(\sigma^2+(1+(I-1)\,\bar{\rho}))} = \\
&= \frac{(1-\rho)}{(1-\bar{\rho}+\sigma^2)}\left[1+\frac{\rho\sigma^2+(I-1)\,\rho\,(1-\rho)+(I-1)\rho^2\sigma^2-(1-\rho)\,(I-1)\,\rho}{(1-\rho)\,(\sigma^2+(1+(I-1)\,\bar{\rho}))}\right] = \psi^p
\end{aligned}$$

where ψ^p *is defined by* $Var(\theta_i|s_i, p^*) = (1-\psi^p)\,\sigma_\theta^2$ *and, hence, the posterior variances coincide.*

Proof 5 ***Proposition 4*** *(*Informational Impact*) Applied twice, the projection theorem gives* $V(\theta_i|s_i)$ *and* $V(\theta_i|s_i,p^*)$*, from which price informativeness* ψ^+ *is derived,*

$$\psi^+ = \frac{\sigma^2\bar{\rho}^2}{(1-\gamma)\,(\sigma^2+1)^2 - \bar{\rho}^2 + \gamma\bar{\rho}\,(\sigma^2+1)}. \tag{53}$$

For any ψ^+ *and* γ*, condition* (53) *is quadratic in* $\bar{\rho}$ *with roots equal*

$$\bar{\rho} = \frac{(\sigma^2+1)}{2\left(\sigma^2+\psi^+\right)}\left[\psi^+\gamma \pm \sqrt{\psi^{+2}\gamma^2 + 4\psi^+\,(1-\gamma)\left(\sigma^2+\psi^+\right)}\right]. \tag{54}$$

For any $\psi^+ \in [0,1]$ *and* $\gamma \in [0,1)$, (54) *gives the values of* $\bar{\rho}$ *that, jointly with* γ*, give rise to price informativeness equal to* ψ^+*. For* $\psi^+ > 0$*, equation* (54) *has a positive and a negative root. In addition, positive (negative) root is decreasing (increasing) in* γ *and increasing (decreasing) in* ψ^+*. Thresholds* $\tau^+_{\gamma,\bar{\rho}}$ *and* $\tau^-_{\gamma,\bar{\rho}}$ *are determined as the changes of* $\bar{\rho}$ *that preserve the same price informativeness with an additional bidder, the inclusion of whom changes* γ *by*

$$\Delta\gamma = \frac{1}{I\,(I-1)} = \frac{(1-\gamma)^2}{2-\gamma}. \tag{55}$$

Using (54)*, the upper threshold is given by*

$$\begin{aligned}
\tau^+_{\gamma,\bar{\rho}} \quad = \quad & \frac{(\sigma^2+1)}{2\left(\sigma^2+\psi^+\right)}[\psi^+\Delta\gamma \\
& +\sqrt{\psi^{+2}\left(\gamma+\Delta\gamma\right)^2 + 4\psi^+\left(1-\gamma-\Delta\gamma\right)\left(\sigma^2+\psi^+\right)} \\
& -\sqrt{\psi^{+2}\gamma^2 + 4\psi^+\,(1-\gamma)\left(\sigma^2+\psi^+\right)}].
\end{aligned} \tag{56}$$

By the monotonicity of (54) *in* γ*,* $\tau^+_{\gamma,\bar{\rho}} < 0$*. The threshold* $\tau^-_{\gamma,\bar{\rho}}$ *can be derived analogously.*

Proof 6 ***Corollary 1*** *(*Price Informativeness in Large Markets*) Using* (53)*, in large markets,*

$$\lim_{\gamma\to 1}\psi^{+}=\frac{\sigma^{2}\bar{\rho}^{2}}{\bar{\rho}\left(\sigma^{2}+1\right)-\bar{\rho}^{2}}. \tag{57}$$

Given that $\sigma^2>0$, $\lim_{\gamma\to 1}\bar{\rho}(\gamma)=0$ *if, and only if,* $\lim_{\gamma\to 1}\psi^{+}=0$*;* $\lim_{\gamma\to 1}\bar{\rho}(\gamma)=1$ *if, and only if,* $\lim_{\gamma\to 1}\psi^{+}=1$*;* $\lim_{\gamma\to 1}\bar{\rho}(\gamma)\in(0,1)$ *if, and only if,* $\lim_{\gamma\to 1}\psi^{+}\in(0,1)$*.*

Proof 7 ***Lemma 2*** *(*Identification 1*) Let* $\{\theta_i\}_{i=1}^{\infty}$ *be a normally distributed random vector. (Only if) Assume that there exists a random variable* X *such that* $R_i=\theta_i-X$ *for all* i*, and* X *and* $\{R_i\}_{i\in I}$ *are pairwise independent. For any* $i\neq j$, $cov(\theta_i,\theta_j)=cov(X,X)=Var(X)$ *is independent of* i,j*, and hence correlations among* $\{\theta_i\}_{i=1}^{\infty}$ *are uniform. (If) Consider a profile* $\{\theta_i\}_{i=1}^{\infty}$ *such that* $cov(\theta_i,\theta_j)=\bar{\rho}$ *for all* $i\neq j$*. To argue existence of a common value component from Definition 2, we show that* $X\equiv\bar{\theta}^{\infty}$ *and* $R_i\equiv\theta_i-X$ *satisfy conditions (i) and (ii). For any vector* $\{\theta_i\}_{i=1}^{I}$ *of the first* $I<\infty$ *elements of* $\{\theta_i\}_{i=1}^{\infty}$, $cov(\theta_j,\theta_i)=\bar{\rho}\sigma_\theta^2$ *for all* $i\neq j$*, and*

$$\begin{aligned}
cov(\bar{\theta},\theta_i-\bar{\theta}) &= \frac{1}{I}\sum_{j\in I}cov(\theta_j,\theta_i)-\frac{1}{I^2}\sum_{j\in I}\sum_{k\in I}cov(\theta_j,\theta_k)=\\
&= \frac{1}{I}\sigma_\theta^2+\frac{I-1}{I}\bar{\rho}-\frac{1}{I^2}\sum_{j\in I}\left(\sigma_\theta^2+(I-1)\,\bar{\rho}\sigma_\theta^2\right)=0.
\end{aligned}$$

Since $\bar{\theta}^{\infty}=\lim_{I\to\infty}\bar{\theta}$, $\lim_{I\to\infty}cov(\bar{\theta},\theta_i-\bar{\theta})=cov(X,R_i)=0$*. Since* $X=\bar{\theta}^{\infty}$ *is normally distributed,* X *and* R_i *are independent. By the assumption of* $cov(\theta_i,\theta_j)=\bar{\rho}\sigma_\theta^2$ *for any* $i\neq j$*, and using that* $cov(\theta_i,\theta_j)=cov(X,X)+cov(R_i,R_j)$ *with* $cov(X,X)=\bar{\rho}\sigma_\theta^2$*, it follows that* $cov(R_i,R_j)=0$ *and, hence,* R_i *and* R_j *are independent.*

For the uniqueness of the decomposition, observe that for an arbitrary common value component X*,*

$$\bar{\theta}^{\infty}\equiv\lim_{I\to\infty}\frac{1}{I}\sum_{i\in I}\theta_i=X+R. \tag{58}$$

For any $I<\infty$*,*

$$Var\left(\frac{1}{I}\sum_{i\in I}R_i\right)=\frac{1}{I^2}\sum_{i\in I}\sum_{j\in I}cov\left(R_i,R_j\right)=\frac{1}{I}Var\left(R_i\right)\leq\frac{1}{I}\sigma_\theta^2. \tag{59}$$

It follows that $Var(R)=0$ *as* $I\to\infty$*; that is,* R *is deterministic.*

Proof 8 ***Lemma 3*** *(*Identification 2*) Let* $\{\theta_i\}_{i=1}^{\infty}$ *be a normally distributed random vector. (Only if) Assume that there exists a random variable* X *such that, for all* i, $\theta_i=$

$X+R_i$, X is independent from R_i for all i, and R_i is independent from R. Then,

$$\begin{aligned}
\lim_{I\to\infty}\frac{1}{I-1}\sum_{j\in I;\ j\neq i} cov(\theta_i,\theta_j) &= cov(\theta_i, X+\lim_{I\to\infty}\frac{1}{I-1}\sum_{j\in I, j\neq i}R_j) = \qquad (60)\\
&= cov(X+R_i, X+\lim_{I\to\infty}\frac{I}{I-1}\frac{1}{I}\sum_{j\in I}R_j - \lim_{I\to\infty}\frac{1}{I-1}R_i) =\\
&= cov(X+R_i, X+R-\lim_{I\to\infty}\frac{1}{I-1}R_i) =\\
&= cov(X+R_i, X+R) = Var\,(X) \equiv \bar{\rho}^{\infty}\sigma_\theta^2. \qquad (61)
\end{aligned}$$

The average correlation $\bar{\rho}^{\infty}$ is independent across traders, and the auction is equicommonal. (If) Consider a large equicommonal auction. We show that $X\equiv\bar{\theta}^{\infty}$ and $R_i\equiv\theta_i - X$ satisfy conditions (i) and (ii) from Definition 3. For any vector $\{\theta_i\}_{i=1}^{I}$ of the first $I<\infty$ elements of $\{\theta_i\}_{i=1}^{\infty}$,

$$\begin{aligned}
cov(\bar{\theta},\theta_i-\bar{\theta}) &= \frac{1}{I}\sum_{j\in I}cov(\theta_j,\theta_i) - \frac{1}{I^2}\sum_{j\in I}\sum_{k\in I}cov(\theta_j,\theta_k) =\\
&= \frac{1}{I}\sigma_\theta^2 + \frac{1}{I}\sum_{j\in I, j\neq i}cov(\theta_j,\theta_i) - \frac{1}{I^2}\sum_{j\in I}\left(\sigma_\theta^2 + \sum_{k\in I, k\neq j}cov(\theta_j,\theta_k)\right) =\\
&= \frac{1}{I}\sum_{j\in I, j\neq i}cov(\theta_j,\theta_i) - \frac{1}{I^2}\sum_{j\in I}\left(\sum_{k\in I, k\neq j}cov(\theta_j,\theta_k)\right).
\end{aligned}$$

Taking the limit as $I\to\infty$ and using that, $\lim_{I\to\infty}\sum_{k\in I, k\neq j}cov(\theta_j,\theta_k) = \bar{\rho}^{\infty}\sigma_\theta^2$ for all k, we have that $cov(X,R_i)=0$. Since $X\equiv\bar{\theta}^{\infty}$ is normally distributed, X and R_i are independent. In addition,

$$\begin{aligned}
\bar{\rho}^{\infty}\sigma_\theta^2 &= \lim_{I\to\infty}\frac{1}{I-1}\sum_{j\in I, j\neq i}cov(\theta_i,\theta_j) = \qquad (62)\\
&= cov(X+R_i, X+R) = Var(X) + cov(R_i, R)
\end{aligned}$$

and $Var(X)=\bar{\rho}^{\infty}\sigma_\theta^2$ imply $cov(R_i,R)=0$ and, hence, the normally distributed R_i, R are independent. For the uniqueness of the decomposition, observe that for an arbitrary common value component X,

$$\bar{\theta}^{\infty} \equiv \lim_{I\to\infty}\frac{1}{I}\sum_{i\in I}\theta_i = X+R. \qquad (63)$$

For any $I < \infty$

$$Var\left(\frac{1}{I}\sum_{i\in I} R_i\right) = cov\left(\frac{1}{I}\sum_{i\in I} R_i, \frac{1}{I}\sum_{j\in I} R_j\right) = \frac{1}{I}\sum_{i\in I} cov\left(R_i, \frac{1}{I}\sum_{j\in I} R_j\right). \tag{64}$$

Since $cov\left(R_i, \lim_{I\to\infty}\frac{1}{I}\sum_{j\in I} R_j\right) = 0$, taking the limit as $I \to \infty$ gives $Var(R) = 0$. It follows that R is deterministic.

Proof 9 ***Lemma 4*** *From the projection theorem, for any y, the vector of coefficients in the conditional expectation $E(y|s_i, p^*)$ is the product*

$$(c_{s_i}, c_{p,y}) = (cov(y, s_i), cov(y, p^*))\,\mathcal{V}^{-1}, \tag{65}$$

where $\mathcal{V}$ is the variance-covariance matrix of vector (s, p^*). The inverse of $\mathcal{V}$ is given by

$$\mathcal{V}^{-1} = \frac{1}{\det(\mathcal{V})}\begin{pmatrix} Var(p^*) & -cov(s_i, p^*) \\ -cov(s_i, p^*) & Var(s_i) \end{pmatrix}. \tag{66}$$

Using (23), we have that $Var(s_i) = \sum_y \sigma_y^2$ and $cov(y, s_i) = \sigma_y^2$. By (25), $cov(s_i, p^*) = \sum_y \beta_y\sigma_y^2$ and $cov(y, p^*) = \beta_y\sigma_y^2$. Using (65),

$$\begin{aligned} c_{p,y} &= \frac{1}{\det(\mathcal{V})}\left[\beta_y\sigma_y^2\sum_{y'}\sigma_{y'}^2 - \sigma_y^2\sum_{y'}\beta_{y'}\sigma_{y'}^2\right] = \\ &= \frac{1}{\det(\mathcal{V})}\sum_{y'\neq y}\left(\beta_y - \beta_{y'}\right)\sigma_y^2\sigma_{y'}^2. \end{aligned} \tag{67}$$

Letting $c = 1/\det(\mathcal{V})$ and observing that $c > 0$ by the positive definiteness of $\mathcal{V}$, we obtain (27).

Proof 10 ***Lemma 5*** *(*UNIFORM-PRICE MODEL*) Let $\{q_i(\cdot)\}_{i\in I}$ be an equilibrium in a uniform price model with a correlation matrix $\mathcal{C}$. In the first-order condition that determines $q_i(\cdot)$ for each i, $\mathcal{C}$ enters only through expectation $E(\theta_n|s_i, p^*)$. By the assumptions of linearity and symmetry of the equilibrium strategies, the equilibrium bid can be written as*

$$q_i(p) = \alpha_0 + \alpha_s s_i + \alpha_p p. \tag{68}$$

Market clearing implies that the equilibrium price is a deterministic function of $\bar{s}$ and, hence, price is informationally equivalent to $\bar{s}$ and $E(\theta_n|s_i, p^*) = E(\theta_n|s_i, \bar{s})$. Mimicking the argument for a double auction, $E(\theta_n|s_i, \bar{s})$ is invariant to changes of $\mathcal{C}$ other than $\bar{\rho}$

and is the same for all equicommonal correlation matrices with commonality $\bar{\rho}$. It follows that $q_i(p)$ satisfies each trader's first-order condition for $\mathcal{C}'$, as well.

Proof 11 ***Corollary 2*** *(*Information Aggregation in Uniform-Price Models*)* *In the class of uniform-price models, price is informationally equivalent to $\bar{s}$ and the results hold by arguments analogous to those in a double auction.*

Lemma 6 *If X is non-degenerate, $\beta_X > \beta_\varepsilon > 0$; $\beta_R \in (0, \beta_\varepsilon)$ if $\bar{\rho}_R < 0$, $\beta_R \in (\beta_\varepsilon, \beta_X]$ if $\bar{\rho}_R > 0$, and $\beta_R = \beta_\varepsilon$ if $\bar{\rho}_R = 0$.*

Proof 12 ***Lemma 6*** *For inequality $\beta_X > \beta_\varepsilon > 0$, using that $cov(X, \bar{s}) = \sigma_X^2$ and $cov(\varepsilon_i, \bar{s}) = \frac{1}{I}\sigma_\varepsilon^2$, we have that $\beta_X = \partial p^*/\partial\bar{s} > \frac{1}{I}\partial p^*/\partial\bar{s} = \beta_\varepsilon > 0$. Since*

$$cov(R_i, \bar{s}) = \frac{1}{I}\left[Var(R_i) + \sum_{j\in I, j\neq i} cov(R_i, R_j)\right], \tag{69}$$

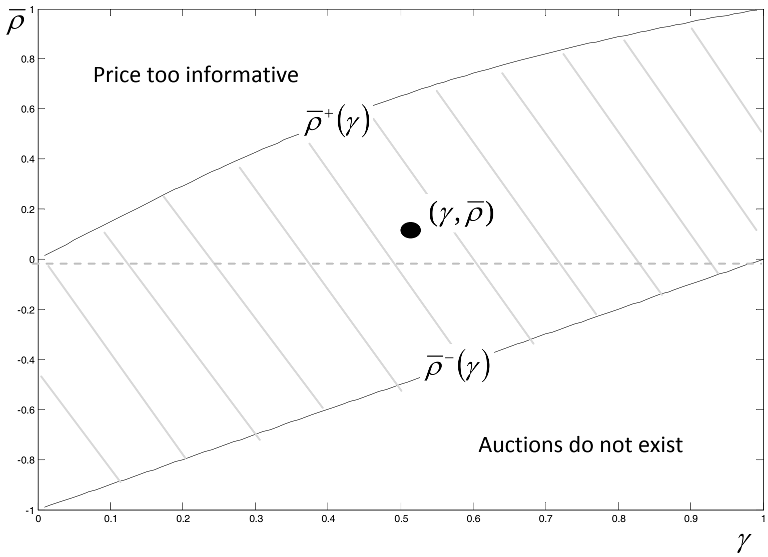
and

$$\begin{aligned} 0 &\leq Var\left(\frac{1}{I}\sum_{i\in I} R_i\right) = \frac{1}{I^2}\left(\sum_{k\in I} Var(R_k) + \sum_{k\in I}\sum_{j\in I, j\neq i} cov(R_k, R_j)\right) \\ &= \frac{1}{I}\left(Var(R_i) - Var(X) + \sum_{j\in I, j\neq i} cov(R_i, R_j) - Var(X)\right) \leq \\ &\leq \frac{1}{I}\left(Var(R_i) + \sum_{j\in I, j\neq i} cov(R_i, R_j)\right), \end{aligned} \tag{70}$$

it follows that $cov(R_i, \bar{s}) \geq 0$ and, hence, $\beta_R = \partial p^*/\partial\bar{s} \times cov(R_i, \bar{s})/Var(R_i) \geq 0$. In addition, (69) *implies that $\beta_R < \beta_\varepsilon$ if, and only if, $\sum_{j\in I, j\neq i} cov(R_i, R_j) < 0$ ($\bar{\rho}_R < 0$). Finally, if $cov(R_i, R_j) = Var(R_i)$ (i.e., residuals are perfectly correlated), then $\beta_R = \beta_X$.*

FIGURE 1: EXISTENCE AND COMMONALITY FUNCTION

A.) AUCTION SPACE



B.) COMMONALITY FUNCTIONS

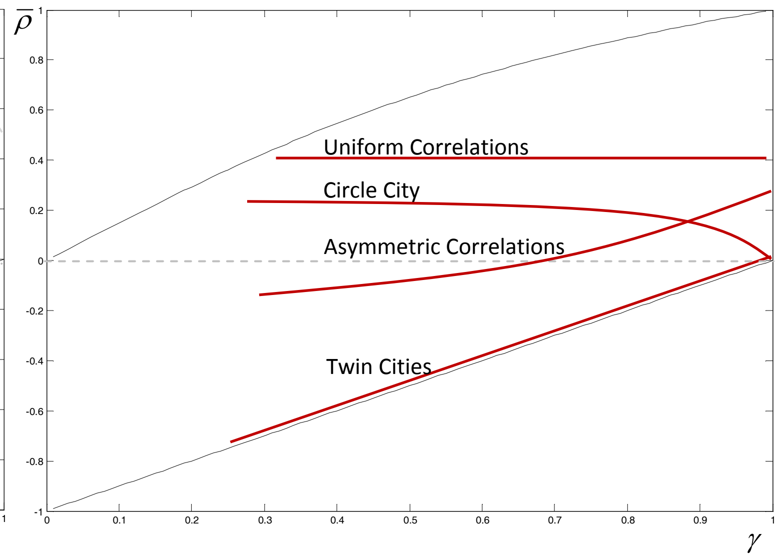
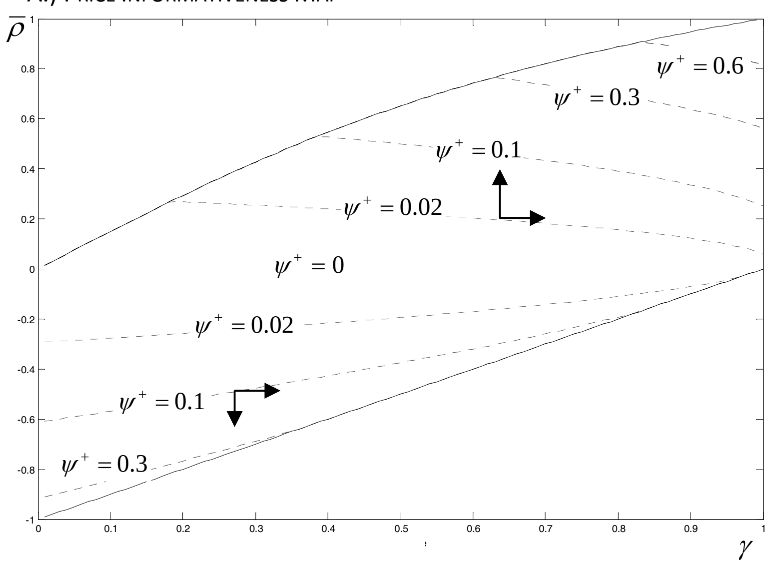


FIGURE 2: PRICE INFORMATIVENESS MAP

A.) PRICE INFORMATIVENESS MAP



B.) INFORMATIVENESS IN EXAMPLES

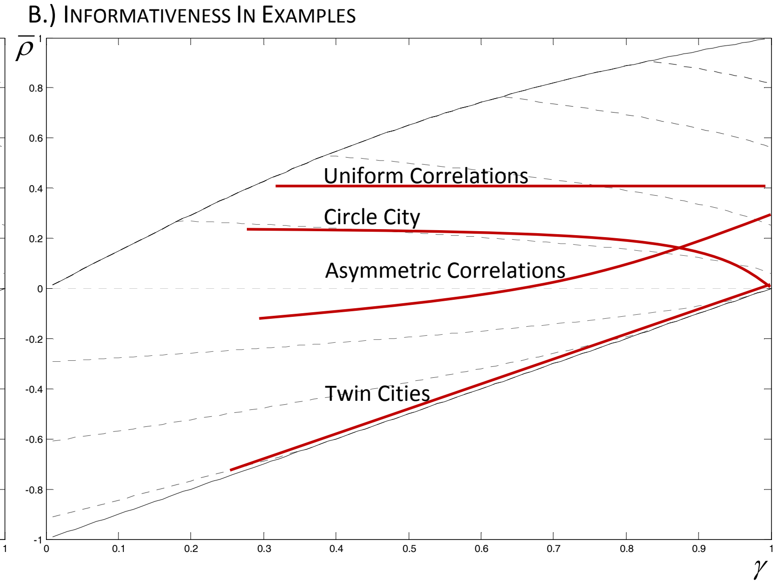
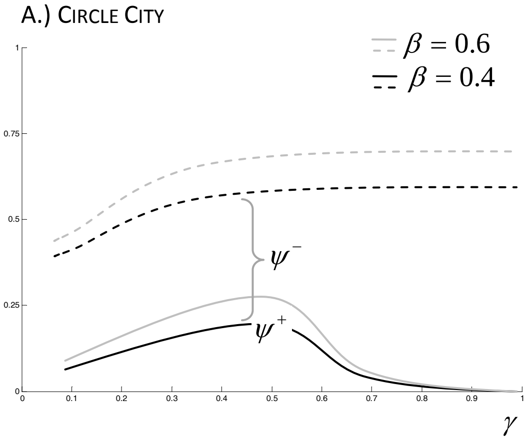
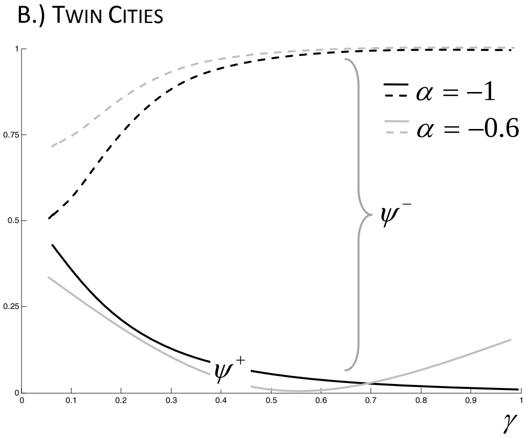


FIGURE 3: PRICE INFORMATIVENESS AND INFORMATIONAL GAP

A.) CIRCLE CITY



B.) TWIN CITIES



C.) ASYMMETRIC CORRELATIONS