Production externalities: Internalization by voting

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Abstract

Internatlization of production externalities is studied in a general equilibrium economy. Production plans are decided by majority voting. Consumers are interested in some degree of internalization depending on their portfolios because they care about dividends of their portfolios rather than dividends of firms. The shareholder governance (one share, one vote) and the stakeholder democracy (one stakeholder, one vote) are compared with respect to internalization of externalities. In general the stakeholder democracy performs better that the shareholder governance.

Keywords: General equilibrium, majority voting, production externalities, shareholder governance vs. stakeholder democracy, social choice.

JEL-classification: D21, D51, D72, G39, L21.

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1 Introduction

In economies some decisions, such as decisions on consumption, saving and work, are made individually while other decisions, such as decisions on production and policy, are made collectively. In case of competitive markets each of the two types of decisions improves the performance of the other type of decisions: 1. utility maximization results in equalization of marginal rates of substitution across consumers so consumers agree unanimously on profit maximization as the aim for collective decisions; 2. profit maximization results in equalization of technical rates of substitution; and, 3. the outcomes are Pareto optimal. However in case of market failures such as externalities, public goods and imperfect competition, the outcomes are not necessarily Pareto optimal.

In the present paper the interaction between individual and collective decisions is studied in a general equilibrium model with competitive markets and externalities between firms. Since the performance of markets with profit maximizing firms is not efficient in presence of externalities, different kinds of regulation such as Pigovian taxes have been considered. However profit maximization is typically not in the interest of the consumers. Indeed consumers typically have conflicting interests: at one extreme a consumer with shares in only one firm wants that firm to maximize its own profit corresponding to no internalization; and, at the other extreme a consumer with the average portfolio wants every firm to maximize aggregate profit of the production sector corresponding to perfect internalization[1]. The argument is illustrated in Hansen & Lott (1996).

Suppose that production plans are decided by majority voting such that a production plan is stable if and only if no other productiton plan is supported by a majority of consumers. Then questions like *who is voting* and *how does*

[1]In models with representative consumers such as most macroeconomic models as well as models, where every agent has some fraction of the market portfolio such as the CAPM model of finance, all consumers agree that the aggregate profit of the production sector should be maximized.

voting perform with respect to internalization of externalities become central. Two governances are considered: the shareholder governance (one share, one vote); and, the stakeholder democracy (one stakeholder, one vote), where every consumer is allowed to vote in every firm. In general every consumer wants firms to maximize a weighted sum of profits where the weights are the shares of the consumer. If there are more than two firms the median voter theorem does not apply. Therefore in order to ensure existence of stable production plans the rate of majority typically has to be larger than one half. In equilibrium consumers maximmize utility, production plans are stable and markets clear.

The relation between the distribution of shares and the performance of the economy is analyzed. Concerning existence of equilibrium it is shown that if the rate of majority is at least $(n-1)/n$, where n is the number of firms, then an equilibrium exists. Concerning internalization of externalities it is shown that if the market portfolio is in the cone of the portfolios of every coalition of consumers greater than the majority rate, then there exists an equilibrium with perfect internalization.

In general, a majority of shareholders in some firm tend to have more shares in that firm than in other firms, so they tend to put "too much" weight on the profit of that firm when voting over production plans. Therefore perfect internalization is typically not supported by the shareholder governance. In the stakeholder democracy the sets of voters in all firms are identical and the average voter has the same number of shares in every firm. Hence perfect internalization is more likely to occur. At first sight the stakeholder democracy appears unrealistic. However public regulation in democracies is a proxi of the stakeholder democracy.

In sports leagues, where cross ownership is prohibited for obvious reasons, the bodies of the leagues, where all clubs often have identical voting weights, aim at internalization of externalities. This is done through sharing broadcast revenue (especially for european soccer leagues) and matchday revenue as well as controlling the distribution of incoming talent (especially for american leagues). In contrast since spectators typically do not vote in the bodies of the leagues there is no internalization of pecuniary externalities between the leagues and the spectators. Therefore sports leagues should be expected to behave like monopolists. See Fort & Quirk (1995), Vroman (1995) and Whitney (2005) for more on sports leagues.

Another example, where markets leads to internalization of externalities, is venture capital in Silicon Valley. In Saxenian (1994) it is documented that there is a substantial degree of information sharing across entrepreneurial firms. In Aoki (2000) it is suggested that venture capitalists ensure information sharing between entrepreneurial firms through cross ownership.

The present paper is organized as follows: in Section 2, the set-up including assumptions and definition of equilbrium is presented; in Sections 3 and 4 the results on existence of equilibrium and internalization of externalities in equilibrium are stated and established; in Section 5 the shareholder governance and the stakeholder democracy are compared with respect to the performance of markets; and, finally in Section 6 some concluding remarks are offered.

2 The model

Set-up

Consider an economy with ℓ goods, m consumers and n firms.

Let $p = (p_1, \ldots, p_\ell)$, where $p_k > 0$ for all k, be a price vector. Price vectors are normalized such that their coordinates sum to one. Let $S = \{\, v \in \mathbb{R}^\ell \,|\, v_k \geq 0$ for all k and $\sum_k v_k = 1 \,\}$ be the set of normalized prices.

Consumers are characterized by their identical consumption sets $X = \mathbb{R}^\ell$, endowment vectors $\omega_i \in \mathbb{R}^\ell$, utility functions $u_i : X \to \mathbb{R}$ and portfolios $\delta_i = (\delta_{i1}, \ldots, \delta_{in})$ where $\delta_{ij} \geq 0$ for all i and j and $\sum_i \delta_{ij} = 1$ for all j.

There are direct externalities between firms: in every firm an action is taken and the production plan of every firm depends on the actions taken in all firms. Firms are described by their sets of action $A_j \subset \mathbb{R}^q$ and production functions $f_j : \prod_{j'} A_{j'} \to \mathbb{R}^\ell$ such that if $a = (a_1, \ldots, a_n)$, where $a_j \in A_j$ for all j, is a list of individual actions, then $y_j = f_j(a)$ is the production plan of firm j. Actions could include choice of some inputs or outputs. As an example to fix ideas suppose that firms choose inputs and that output in every firm depends on aggregate inputs: if capital $K_j \geq 0$ is chosen in firm j, then the production plan of firm j is $(-K_j, (\sum_{j' \neq j} K_{j'})^\alpha K_j^\beta)$ where $\alpha, \beta > 0$.

In the traditional approach to externalities in general equilibrium firms are described by correspondences $Y_j : (\mathbb{R}^\ell)^{n-1} \to \mathbb{R}^\ell$ such that if $y_{-j} = (y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n)$ is a list of individual production plans for all firms but firm j, then the production set of firm j is $Y_j(y_{-j})$. However the traditional approach is adequate to study of internalization of externalities. Indeed if the production plan of firm j is changed, then the production set of firm j' changes too. Therefore it might be that the production plan of firm j' is not possible anymore. Alternatively it might be that other, more attractive production plans become available. Hence if the production plan of firm j is changed, then the production plan of firm j' might change too. Thus in the traditional approach if consumers are considering to change the production plan of firm j, then they need to have conjectures or form expectations about how the production plans of the other firms will change. The approach of the present paper is more adequate as it eliminates the need to introduce and consider conjectures and expectations.

Demand, supply and equilibrium

For a price vector p and a list of individual actions $a = (a_1, \ldots, a_n)$, where $a_j \in A_j$ for all j, the problem of consumer i is

$$\begin{aligned} \max_{x_i} \quad & u_i(x_i) \\ \text{s.t.} \quad & p \cdot x_i \leq p \cdot \omega_i + \textstyle\sum_j \delta_{ij} p \cdot f_j(a). \end{aligned}$$

The problem of firm j takes a few steps. For a price vector p and a list of individual actions a let $P_{ij}(p, a) \subset A_j$ be the set of actions in firm j that make consumer i better off than a_j so

$$P_{ij}(p,a) \;=\; \{\, a'_j \in A_j \,|\, \textstyle\sum_{j'} \delta_{ij'} p \cdot f_{j'}(a'_j, a_{-j}) > \sum_{j'} \delta_{ij'} p \cdot f_{j'}(a) \,\}.$$

For a price vector p, a list of individual actions a and another action a'_j for firm j let $M_j(p,a,a'_j) \subset \{1,\ldots,m\}$ be the set of consumers who are better off with a'_j than with a_j so

$$M_j(p,a,a'_j) \;=\; \{\, i \in \{1,\ldots,m\} \,|\, a'_j \in P_{ij}(p,a) \,\}.$$

Let $\rho \in [0,1]$ be the rate of majority needed to change actions in firms and let $\theta = (\theta_1,\ldots,\theta_n)$, where $\theta_j = (\theta_{1j},\ldots,\theta_{mj})$ and $\theta_{ij} \geq 0$ and $\sum_i \theta_{ij} = 1$, be the voting weights. For a change of actions from a_j to a'_j in firm j the change wins if and only if $\sum_{i \in M_j(p,a,a'_j)} \theta_{ij} > \rho$. Two cases of voting weights are considered: the shareholder governance where $\theta_{ij} = \delta_{ij}$ (one share, one vote); and, the stakeholder democracy where $\theta_{ij} = 1/m$ (one stakeholder, one vote).

For a price vector p and a list of individual actions a, let $Q^\rho_j(p,a) \subset A_j$ be the set of actions preferred to a_j in firm j so

$$Q^\rho_j(p,a) \;=\; \{\, a'_j \in A_j \,|\, \textstyle\sum_{i \in M_j(p,a,a'_j)} \theta_{ij} > \rho \,\}.$$

For a price vector p and a list of individual actions a_{-j} for all firms but firm j, the problem of firm j is to find an action a_j such that $Q^\rho_j(p,a_j,a_{-j}) = \emptyset$.

Definition 1 *An* **equilibrium** *is a price vector, a list of individual consumption bundles and a list of individual actions* $(\bar{p},\bar{x},\bar{a})$ *such that:*

(C) $\bar{x}_i$ *is a solution to the problem of consumer* i *given* $\bar{p}$ *and* $\bar{a}$ *for all* i*.*

(F) $\bar{a}_j$ *is a solution to the problem of firm* j *given* $\bar{p}$ *and* $\bar{a}_{-j}$ *for all* j*.*

(E) $\sum_i \bar{x}_i = \sum_i \omega_i + \sum_j f_j(\bar{a})$*.*

Internalization

Consider an equilibrium $(\bar{p}, \bar{x}, \bar{a})$. In case of *no internalization*, where every firm maximizes its own profit, the profit maximization problem of firm j is

$$\begin{aligned} \max_{a_j} \quad & \bar{p} \cdot f_j(a_j, \bar{a}_{-j}) \\ \text{s.t.} \quad & a_j \in A_j. \end{aligned}$$

Typically the equilibrium allocation is not Pareto optimal.

In case of *perfect internalization*, where every firm maximizes aggregate profit, the profit maximization problem of firm j is

$$\begin{aligned} \max_{a_j} \quad & \sum_{j'} \bar{p} \cdot f_{j'}(a_j, \bar{a}_{-j}) \\ \text{s.t.} \quad & a_j \in A_j. \end{aligned}$$

The equilibrium allocation is Pareto optimal.

3 Results

Assumptions

Consumers are supposed to satisfy the following assumptions:

(A.1) $u_i \in C(X, \mathbb{R})$.

(A.2) u_i is strictly monotone, so $z_i^k \geq x_i^k$ for all k and $z_i \neq x_i$ imply that $u_i(z_i) > u_i(x_i)$, and quasi-concave, so $u_i(z_i), u_i(z'_i) \geq u_i(x_i)$ and $z'_i \neq z_i$ imply that $(1-\tau)u_i(z_i) + \tau u_i(z'_i) \geq u_i(x_i)$ for all $\tau \in [0, 1]$.

(A.3) The set $u_i^{-1}(r) = \{\, x_i \in X \,|\, u_i(x) = r \,\}$ is bounded from below for all $r \in \mathbb{R}$.

All assumptions are standard.

The firms are supposed to satisfy the following assumptions:

(A.4) The set of actions A_j is convex and compact.

(A.5) $f_j \in C(\prod_j A_j, \mathbb{R}^\ell)$.

(A.6) f_j is concave, so $f_j^k((1-\tau)y_j + \tau y_j') \geq (1-\tau)f_j^k(y_j) + \tau f_j^k(y_j')$ for all k and $\tau \in [0,1]$.

The assumptions ensure that the comprehensive hulls of the production sets

$$\{\, y \in \mathbb{R}^{\ell n} \,|\, y_{j'}^k \leq f_{j'}^k(a) \text{ for all } j' \text{ and } k \text{ and some } a \in A \,\}$$

are convex.

Existence of equilibrium

The conflicts between voters over the choices of actions can be reformulated as conflicts over the relative weights on profits in firms. Therefore the majority rate needed to ensure existence of equilibrium depends on the number of firms.

Theorem 1 *Suppose that*

$$\rho \;\geq\; \frac{n-1}{n}.$$

Then every economy has an equilibrium.

The proof of Theorem 1 is postponed to the next section. The proof rests on two observations. The first observation is that for a coalition of consumers $M_j \subset \{1,\ldots,n\}$ if the action of firm j is chosen to maximize $\sum_{j'} \delta_{j'} p \cdot j'(a_j, a_{-j})$, where $(\delta_1,\ldots,\delta_n)$ is in the cone of the portfolios of the consumers in M_j, then there is no a_j' such that all consumers in M_j are better off. The second observation is that if $\rho \geq (n-1)/n$, then the intersection of all cones of portfolios for coalitions of consumers with more than ρ votes is non-empty. Therefore if the action of firm j is chosen to maximize $\sum_{j'} \delta_{j'} p \cdot f_{j'}(a_j, a_{-j})$, where $(\delta_1,\ldots,\delta_n)$ is in the intersection of the cones of the portfolios of consumers with more than $\rho \geq (n-1)/n$ votes, then actions are stable.

Internalization in equilibrium

There is perfect internalization if the market portfolio is in the cone of the portfolios of every coalition of consumers with more than the majority rate votes.

Theorem 2 *For an economy suppose that for every firm j and every coalition of consumers M_j with $\sum_{i \in M_j} \theta_{ij} > \rho$ there exists $(\lambda_i)_{i \in M_j}$ with $\lambda_i \geq 0$ for all i such that*

$$\sum_{i \in M_j} \lambda_i \delta_i \; = \; \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Then the economy has an equilibrium with perfect internalization.

The proof of Theorem 2 is postponed to the next section. The proof rests on two observations mentioned after Theorem 1 and a third observation: if the condition of the Theorem is satisfied, then the portfolio $\delta = (1, \ldots, 1)$ is in the intersection of all the cones of portfolios for coalitions of consumers with more than ρ votes. Therefore if the action of firm j is chosen to maximize $\sum_{j'} p \cdot f_{j'}(a_j, a_{-j})$ for all j, then there is perfect internalization and actions are stable.

4 Proofs of Theorems 1 and 2

Following Tvede & Crès (2005) an artificial economy is used. The problem of the firm is decomposed into a profit maximization problem and a selection problem of price vector for profit maximization.

Let $\Delta \subset \mathbb{R}^{\ell n}$ be defined by

$$\Delta \; = \; \{ v \in \mathbb{R}^{\ell n} \,|\, \textstyle\sum_j \sum_k v_j^k = 1 \}.$$

Let $M \subset \{1, \ldots, m\}$ be the set of consumers with shares in some firms so

$$M \; = \; \{ i \in \{1, \ldots, m\} \,|\, \delta_{ij} > 0 \text{ for some } j \}.$$

For $i \in M$ let $d_i \in \Delta$ be the normalized portfolio of consumer i so $d_i = (1/\sum_j \delta_{ij})\delta_i$.

Let the correspondence $V_i : S \times \Delta \to \Delta$ associate every price vector p in S and vector μ in Δ with the set of vectors μ' in Δ closer to

$$\begin{pmatrix} d_{i1}p \\ \vdots \\ d_{in}p \end{pmatrix}$$

than μ, so

$$V_i(p,\mu) \;=\; \{\, \mu' \in \Delta \,|\, \textstyle\sum_j \|\mu'_j - d_{ij}p\|^2 < \sum_j \|\mu_j - d_{ij}p\|^2 \,\}.$$

Let the correspondence $N : S \times \Delta \times \Delta \to M$ associate every price vector p in S and pair of vectors μ and μ' in Δ with the set of consumers with $\mu' \in V_i(p,\mu)$, so

$$N(p,\mu,\mu') \;=\; \{\, i \in M \,|\, \mu' \in V_i(p,\mu) \,\}.$$

Let the correspondence $W_j^\rho : S \times \Delta \to \Delta$ associate price vector p and vector v in Δ with the set of vectors v' in Δ prefered to v, so

$$W_j^\rho(p,\mu) \;=\; \{\mu' \in \Delta \,|\, \textstyle\sum_{i \in N(p,\mu,\mu')} \theta_{ij} > \rho\}$$

Definition 2 *An* **artificial equilibrium** *is a list of individual vectors in Δ, a price vector, a list of individual consumption bundles and a list of individual actions $(\bar\mu, \bar p, \bar x, \bar a)$ such that:*

(C) $\bar x_i$ is a solution to the problem of consumer i given $\bar p$ and $\bar a$ for all i;

(F') $\bar a_j$ maximizes the profit of firm j given $\bar\mu_j$, so $\bar a_j$ is a solution to

$$\begin{aligned} \max_{a_j} \quad & \sum_{j'} \bar\mu_j^{j'} \bar p \cdot f_{j'}(a_j, \bar a_{-j}) \\ s.t. \quad & a_j \in A_j : \end{aligned}$$

(F") $W_j^\rho(\bar p, \bar\mu_j) = \emptyset$; and,

(E) $\sum_i \bar{x}_i = \sum_i \omega_i + \sum_j f_j(\bar{a})$.

The problems of the firms in Definition 2 are artificial in the sense that they are not related to the preferences of the consumers. However as shown in Lemma 1 if $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium, then $(\bar{p}, \bar{x}, \bar{a})$ is a equilibrium.

Lemma 1 *Suppose that $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium, then $(\bar{p}, \bar{x}, \bar{a})$ is an equilibrium.*

Proof: Suppose that $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium. Then (C) and (E) are satisfied. Therefore it suffice to show that (F) is satisfied. The strategy of the proof is to show that if $Q_j^\rho(\bar{p}, \bar{a}) \neq \emptyset$, then $W_j^\rho(\bar{p}, \bar{\mu}) \neq \emptyset$.

Suppose that $Q_j^\rho(\bar{p}, \bar{a}) \neq \emptyset$, then there exists j and $a_j \in A_j$ such that

$$\sum_{i \in M_j(\bar{p}, \bar{a}, a_j)} \theta_{ij} > \rho$$

Let $\bar{y}_{j'} = f_{j'}(\bar{a})$ and $y_{j'} = f_{j'}(a_j, \bar{a}_{-j})$ for all j', then

$$\sum_{j'} \bar{\mu}_j^{j'} \cdot (y_{j'} - \bar{y}_{j'}) \ \leq \ 0$$

and

$$\sum_{j'} d_{ij'} \bar{p} \cdot (y_{j'} - \bar{y}_{j'}) \ > \ 0$$

for all $i \in M_j(\bar{p}, \bar{a}, a_j)$.

For $e \in \mathbb{R}^\ell$ defined by

$$e \ = \ \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

let $\mu_j \in \Delta$ be defined by

$$\mu_j \ = \ \bar{\mu}_j + \tau \left(\begin{pmatrix} y_1 - \bar{y}_1 \\ \vdots \\ y_n - \bar{y}_n \end{pmatrix} - \frac{\sum_{j'} (y_{j'}^k - \bar{y}_{j'}^k) \cdot e}{\ell n} \begin{pmatrix} e \\ \vdots \\ e \end{pmatrix} \right).$$

Then $\mu_j \in V_i(\bar{p}, \bar{\mu}_j)$ if and only if

$$\sum_{j'} (2d_{ij'}\bar{p} - (\mu_j^{j'} + \bar{\mu}_j^{j'})) \cdot (\mu_j^{j'} - \bar{\mu}_j^{j'}) \ > \ 0.$$

Therefore there exists $\tau > 0$ such that $\mu_j \in V_i(\bar{p}, \bar{\mu}_j)$ for all $j \in M_j(\bar{p}, \bar{a}, a_j)$ because

$$\sum_{j'} (2d_{ij'}\bar{p} - (\mu_j^{j'} + \bar{\mu}_j^{j'})) \cdot (\mu_j^{j'} - \bar{\mu}_j^{j'}) \ = \ 2\tau \sum_{j'} (d_{ij'}\bar{p} - \bar{\mu}_j^{j'}) \cdot (y_{j'} - \bar{y}_{j'})$$

$$-\tau^2 \sum_{j'} \left\| (y_{j'} - \bar{y}_{j'}) - \frac{\sum_{j''} (y_{j''} - \bar{y}_{j''}) \cdot e}{\ell n} e \right\|^2 .$$

Hence if $Q_j^\rho(\bar{p}, \bar{a}) \neq \emptyset$ then $W_j^\rho(\bar{p}, \bar{\mu}) \neq \emptyset$. Thus if $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium, then $(\bar{p}, \bar{x}, \bar{a})$ is an equilibrium.

□

For $p \in \mathbb{R}^\ell_{++}$ let $\pi_i(p)$ in Δ for consumer i be defined by

$$\begin{pmatrix} d_{i1}p \\ \vdots \\ d_{in}p \end{pmatrix}.$$

Then $\pi_i(p)$ is the ideal point of consumer i in the sense that $V_i(p, \pi_i(p)) = \emptyset$. For a subset of consumers $M_j \subset M$ let $\mathrm{co}\{\pi_i\}_{i \in M_j}$ be the convex hull of the ideal points for the consumers in M_j. Clearly if $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium, then $\bar{\mu}_j$ is in the set $\mathrm{co}\{\pi_i\}_{i \in M}$ for all j and the set $\mathrm{co}\{\pi_i\}_{i \in M}$ has dimension $n - 1$. For firm j and rate of majority ρ let $\mathcal{M}_j^\rho \subset 2^m$ be the family of subsets of consumers in M with voting weight greater than ρ, so $M_j \subset \mathcal{M}_j^\rho$ if and only if $\sum_{i \in M_j} \theta_{ij} > \rho$.

Proof of Theorem 1

According to Greenberg (1979) if $\rho \geq (n-1)/n$, then $\cap_{M_j \in \mathcal{M}_j} \mathrm{co}\{\pi_i\}_{i \in M_j} \neq \emptyset$. Suppose that $\bar{\mu}_j \in \cap_{M_j \in \mathcal{M}_j} \mathrm{co}\{\pi_i(p)\}_{i \in M_j}$. Then $W_j^\rho(p, \bar{\mu}_j) = \emptyset$ for all $p \in S$ so (F") is satified for all $p \in S$.

For p and a_{-j} the problem of the firm j is

$$\begin{array}{ll} \max_{a_j} & \sum_{j'} \mu_j^{j'} p \cdot f_{j'}(a_j, a_{-j}) \\ \text{s.t.} & a_j \in A_j. \end{array}$$

It follows from Berge's maximum theorem that the solution correspondence $\alpha_j : S \times A_{-j} \to A_j$ of firm j is upper hemi-continuous and convex valued and the profit function $\psi_j : S \times A_{-j} \to \mathbb{R}$ is continuous.

Let $v_L, v_U \in \mathbb{R}^\ell$ be such that if $a_j \in \alpha_j(p, a_{-j})$ for some p and a_{-j}, then $v_L^k < f_j^k(a_j, a_{-j}) < v_U^k$ for all j and k. Let $w \in \mathbb{R}^\ell$ be such that if $u_i(x_i) \geq u_i(\omega_i + \sum_j \delta_{ij} v_j)$, then $x_i^k > w^k$ for all i and k. Let the truncated consumption set $X^T \subset \mathbb{R}^\ell$ be defined by

$$X^T = \{x \in X \,|\, w^k \leq x^k \leq \textstyle\sum_i \omega_i^k + \sum_j v_U^k - (m-1) w^k \text{ for all } k\}.$$

Then for p and a the truncated problem of consumer i is

$$\begin{array}{ll} \max\limits_{x_i} & u_i(x_i) \\ \text{s.t.} & \begin{cases} p \cdot x_i \leq p \cdot \omega_i + \sum\limits_j \psi_j(p, a_{-j}) \\ x_i \in X^T. \end{cases} \end{array}$$

It follows from Berge's maximum theorem that the demand correspondence $\beta_i : S \times \prod_j A_j \to X^T$ is upper hemi-continuous and convex valued.

For x and a the price problem is

$$\begin{array}{ll} \max\limits_{p} & p \cdot \left(\sum\limits_i x_i - \sum\limits_i \omega_i - \sum\limits_j f_j(a) \right) \\ \text{s.t.} & p \in S. \end{array}$$

It follows from Berge's maximum theorem that the price correspondence $\gamma : (X^T)^m \times \prod_j A_j \to S$ is upper hemi-continuous and convex valued.

Let the correspondence $\Gamma : S \times (X^T)^m \times \prod_j A_j \to S \times (X^T)^m \times \prod_j A_j$ be defined by

$$\Gamma(p, x, a) = (\gamma(x, a), \beta_1(p, a), \ldots, \beta_m(p, a), \alpha_1(p, a_{-1}), \ldots, \alpha_n(p, a)).$$

It follows from Kakutani's fixed point theorem that the correspondence has a fixed point $(\bar{p}, \bar{x}, \bar{a})$. Clearly $(\bar{\mu}, \bar{p}, \bar{x}, \bar{a})$ is an artificial equilibrium so $(\bar{p}, \bar{x}, \bar{a})$ is an equilibrium.

Proof of Theorem 2

For every firm j and every coalition of consumers M_j with $\sum_{i \in M_j} \theta_{ij} > \rho$ suppose that there exists $\lambda = (\lambda_1, \ldots, \lambda_m)$ with $\lambda_i \geq 0$ for all i such that

$$\sum_{i \in M_j} \lambda_i \delta_i = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Then

$$\frac{1}{n} \begin{pmatrix} p \\ \vdots \\ p \end{pmatrix} \in \cap_{M_j \in \mathcal{M}_j} \text{co} \{\pi_i(p)\}_{i \in M_j}.$$

Therefore it follows from the proof of Theorem 1 that there exists an equilibrium with perfect internalization.

5 Shareholders or stakeholders?

In the present section the shareholder governance and the stakeholder democracy are compared. First we provide a couple of examples to illustrate the possible outcomes of the shareholder governance and the stakeholder democracy. Second we use parts of the literature on social choice to provide some more general insights.

Some examples

Assumed that there are two firms and that the rate of majority is one half to keep the discussion simple.

Example 1: Suppose that: the distribution of shares is symmetric around the diagonal, so if there exists a consumer with portfolio $\delta = (\delta_1, \delta_2)$, then there exists a consumer with portfolio (δ_2, δ_1); and, there exists a consumer with a portfolio (δ_1, δ_2) where $\delta_2 \neq \delta_1$. The first property reflects that there are no wealth effects in portfolios in the sense that their distribution is independent of their size: it is not the case that consumers with small portfolios tend to have more shares in firm 1 and consumers with large portfolios tend to have more shares in firm 2 or vice versa. The second property reflects a conflict in the sense that at least one consumer wants to put more weight on the profit of firm 1 than on the profit of firm 2 (and at least one other consumer wants to put more weight on the profit of firm 2 than on the profit of firm 1). An example of such a distribution of portfolios is shown in Figure 1 below.

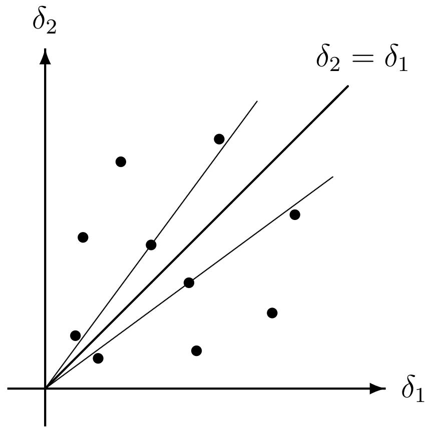

Figure 1: *A distribution of portfolios.*

Consider the weights (λ_1, λ_2) for profit maximization in firm 1. These weights are stable in the voting process if votes are equally distributed below and above the half line through 0 and (λ_1, λ_2). Perfect internalization demands that weights for profit maximization on the diagonal ($\delta_1 = \delta_2$) are stable.

For the shareholder governance, the shareholders with more shares in firm

1 than in firm 2 have more than 50 pct. of the votes in firm 1. Hence there is more than 50 pct. of the votes below the diagonal so the shareholder governance does not support perfect internalization.

For the stakeholder democracy the shareholders with more shares in firm 1 than in firm 2 have 50 pct. of the votes. In equilibrium the relative weights have to be between the two thin lines. Thus the stakeholder democracy does support perfect internalization.

Example 2: Suppose that there are wealth effects. An example of such a distribution of portfolios is shown in Figure 2 below. Suppose that the consumer with the large portfolio has something like 55 pct. of the shares in firm 1 and 45 pct. of the shares in firm 2 and that the three consumers with more shares in firm 2 than in firm 1 have around five times more shares in firm 2 than in firm 1.

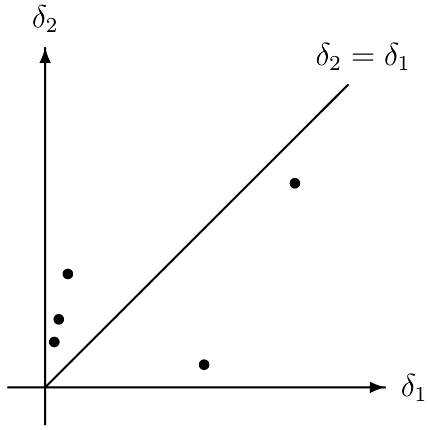

Figure 2: *Another distribution of portfolios.*

In firm 1 the shareholder with a large portfolio has a majority on its own. Therefore in equilibrium for the shareholder governance, the relative weight on the profits in firm 1 is 55/45. Hence internalization is not supported, but a high degree of internalization is supported. In equilibrium for the stakeholder democracy the relative weight on the profits in firm 1 and firm 2 is 1/5. Thus only a very low degree of internalization is supported.

The discussion above is based on a couple of figures, but convey the insight that if there are many small shareholders and no wealth effects, then the stakeholder democracy is likely to support perfect internalization whereas the shareholder governance does not.

Social choice

There are at most m shareholders, where consumer i wants firm j to maximize $p \cdot \sum_{j'} \delta_{ij'} f_{j'}(a)$. Therefore the conflict over the choice of actions $a_j \in A_j$ in firm j can be reformulated as a conflict over the relative weights $(\lambda_{jj'})_{j'}$ with $\lambda_{j1}, \ldots, \lambda_{jn} \geq 0$ and $\sum_{j'} \lambda_{jj'} = 1$ on profits in firms $\max_{a_j \in A_j} p \cdot \sum_{j'} \lambda_{jj'} f_{j'}(a_j, a_{-j})$. The parameters of the conflict over the relative weights are the portfolios $(\delta_i)_i$ of the consumers.

The dimension of the set of relative weights is $n-1$. In Greenberg (1979) the median voter theorem is generalized to multi-dimensional sets and the result is that if the majority rate is at least $(n-1)/n$, then there exists a equilibrium and the equilibrium weights may be considered as the portfolio of the generalized median shareholder. Theorem 1 builds on Greenberg (1979).

With assumptions about the distribution of portfolios it is possible to characterize some of the stable outcomes and lower the super-majority rate needed to ensure existence of equilibrium. Suppose that the distribution of portfolios is symmetric around the market portfolio line (no wealth effects), then according to Grandmont (1978) the average portfolio is stable (as relative weights) for the simple majority rate $\sigma = 0.5$. Next, suppose that there is a continuum of consumers and that the distribution of portfolios is ρ-concave for some $\rho \geq 0$ (if ψ is the density and $(\psi(\delta))^\alpha$ is concave, then $\psi(\delta)$ is α-concave), then according to Caplin & Nalebuff (1991) the average portfolio is stable (as relative weights) for $\sigma \geq 1 - 1/e \approx 0.64$.

Continuing to assume that there is a continuum of consumers and that the distribution of portfolios is α-concave, for the shareholder governance the average portfolio in firm j is $\sum_i \delta_{ij}\delta_i$. Hence unless the group of owners $\{\, i \,|\, \delta_{ij} > 0 \,\}$ of firm j is identical to the group of owners $\{\, i \,|\, \delta_{ik} > 0 \,\}$ of firm k for all pairs of firms j and k, the average portfolios in some firms are not the market portfolio. Thus the shareholder governance does not support perfect internalization. For the stakeholder democracy the average portfolio in firm j is the market portfolio $\sum_i \delta_i$. Therefore the stakeholder democracy does support perfect internalization.

6 Concluding remarks

A couple of natural extensions come into mind: negative amounts of shares; and, externalities between firms and consumers. Firstly for negative amounts of shares, the analysis should carry over except that the rate of super majority needed to ensure existence of equilibrium in Theorem 1 has to be increased to $n/(n+1)$ because relative weights $(\lambda_{jk})_k$ cannot be normalized by $\sum_k \lambda_{jk} = 1$. Secondly for externalities such as pollution of the environment between firms and consumers, externalities are described by preferences of consumers rather than by portfolios. Therefore the dimension of the conflict is m rather than $n-1$ and there is no natural generalization of Theorem 2. However assuming that preferences are intermediate à la Grandmont (1978) seems to be a promising approach to the study of the performance of markets in the presence of externalities between firms and consumers.

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